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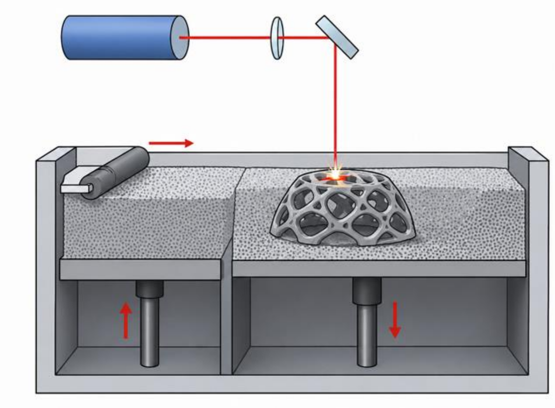

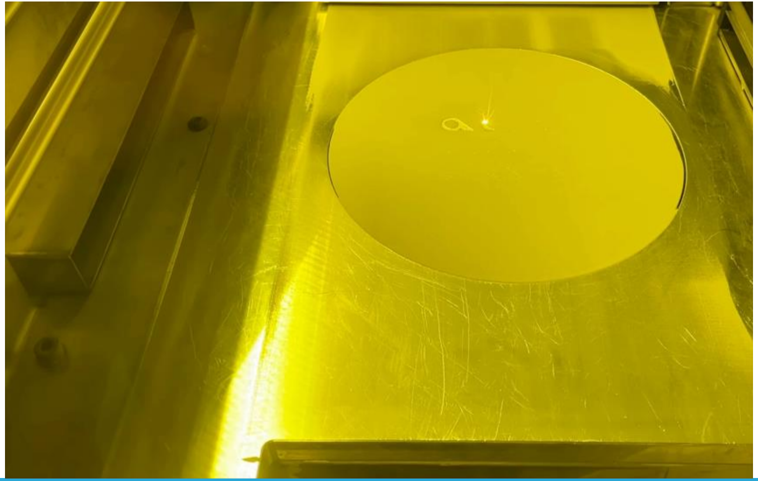

Efficient Thermal Simulation in Metal Additive Manufacturing via Semi-Analytical Isogeometric Analysis

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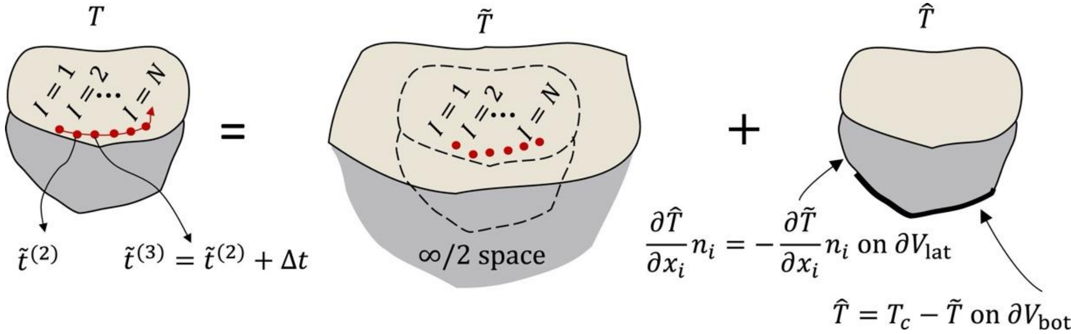
METHOD · ANALYTICAL + NUMERICAL DECOMPOSITION

An analytical + correction split of T



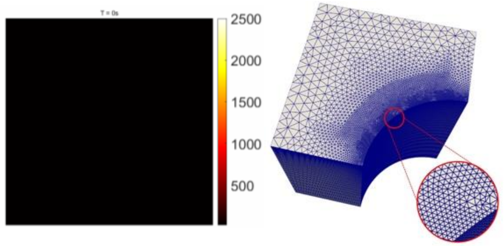
GOVERNING PROBLEM

$\partial T/\partial t = \alpha \nabla^2 T + Q/(\rho c_p)$ in V

$\partial T/\partial n = 0$ on $\partial V_{lat,top}$; $T = T_{con}$ on ∂V_{bot}

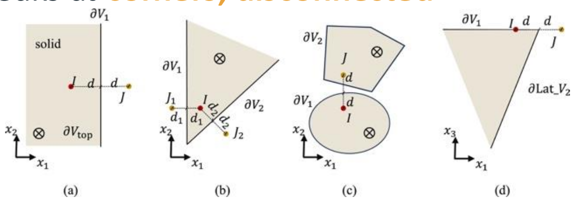
THE BOTTLENECK

LPBF drives steep gradients on the scale of the laser spot, **r ≈ 20 µm**. FEM then needs uniformly fine meshes or scan-wise remeshing — **intractable at part scale**.



WHY IMAGE-SOURCES FAIL

The classical image-source method enforces adiabatic walls only for prismatic blocks. On real CAD it breaks at **corners, disconnected solids, and varying cross-sections**.



OUR DECOMPOSITION

$$T = \tilde{T} + \hat{T}$$

Analytical half-space field

Superposition of instantaneous point-source Green's functions.
Closed-form, mesh-free, evaluated only where it matters.

$$\tilde{T}(\mathbf{x},t)=\sum_{I=1}^{K}\frac{E^{(I)}}{\rho c_p\left[4\pi\alpha(t-\tau^{(I)})\right]^{3/2}}\exp\left(-\frac{\|\mathbf{x}-\mathbf{x}^{(I)}\|^2}{4\alpha(t-\tau^{(I)})}\right)H(t-\tau^{(I)})$$

IGA correction field

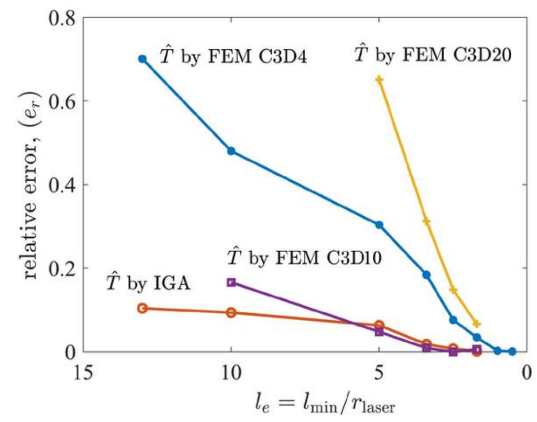
A smooth NURBS solve of the heat equation whose boundary conditions cancel the half-space residual on the true geometry.

$$\rho c_p \frac{\partial \hat{T}}{\partial t} = k\,\nabla^2 \hat{T}, \quad \frac{\partial \hat{T}}{\partial \mathbf{n}} = -\frac{\partial \tilde{T}}{\partial \mathbf{n}} \text{ on } \partial V_{\text{lat}} \cup \partial V_{\text{top}}, \quad \hat{T} = T_c - \tilde{T} \text{ on } \partial V_{\text{bot}}.$$

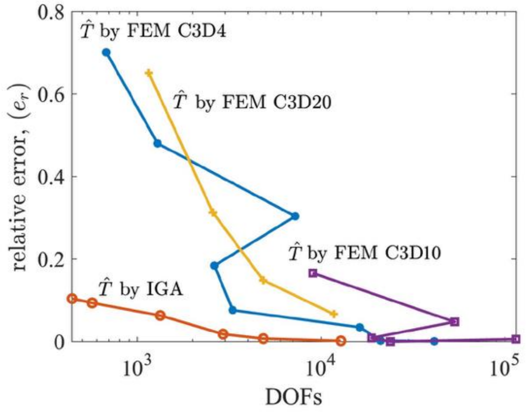
***Heat-source agnostic** — any kernel admitting a half-space solution (Gaussian surface source shown).*

Matched accuracy at a fraction of the cost

Relative L^2-error of the total field T

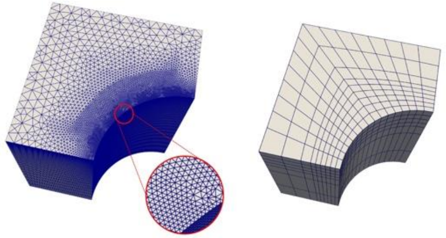


vs element size $l_e = l_{min}/r$



vs degrees of freedom

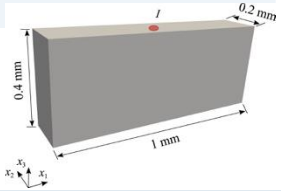
IGA holds **$e_r \lesssim 10\%$** even at $l_e = 13$ (elements 13× the laser radius), and sits **1–2 orders of magnitude** below FEM at matched DOFs.



80×
vs Abaqus C3D4

258×
vs Abaqus C3D20

CPU / step at matched 5% error

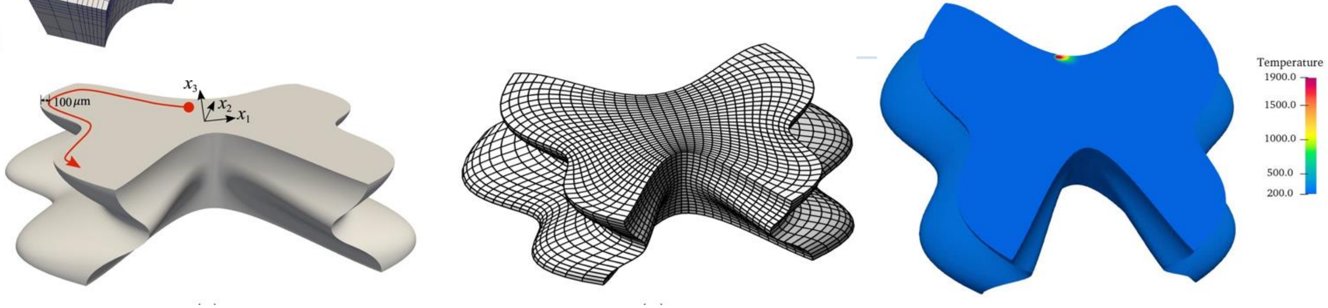


Thin-wall benchmark (uniform mesh, $e_r \le 5\%$)

Method	DOFs	CPU/step
IGA (G+Smo, quad.)	2 639	**0.132 s**
Abaqus C3D4	11 536	10.6 s
Abaqus C3D20	45 261	34.0 s

C^{p-1} NURBS resolve through-thickness gradients smoothly across coarse elements — where C^0 FEM must keep refining l_{min}.

Generalises to free-form CAD: adiabatic BCs enforced on a butterfly part (cubic-NURBS, 27 456 DOFs) where image-sources are inapplicable.



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