Efficient Thermal Simulation in Metal Additive Manufacturing via Semi-Analytical Isogeometric Analysis

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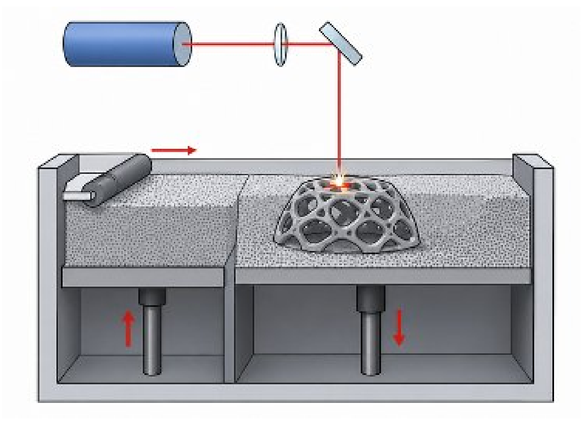
Ye Ji[1], Yang Yang[2], Matthias Möller[1], Can Ayas[2]
[1]Delft Institute of Applied Mathematics · [2]Dept. of Precision and Microsystems Engineering · **Delft University of Technology**, The Netherlands · y.ji-1@tudelft.nl

Introduction

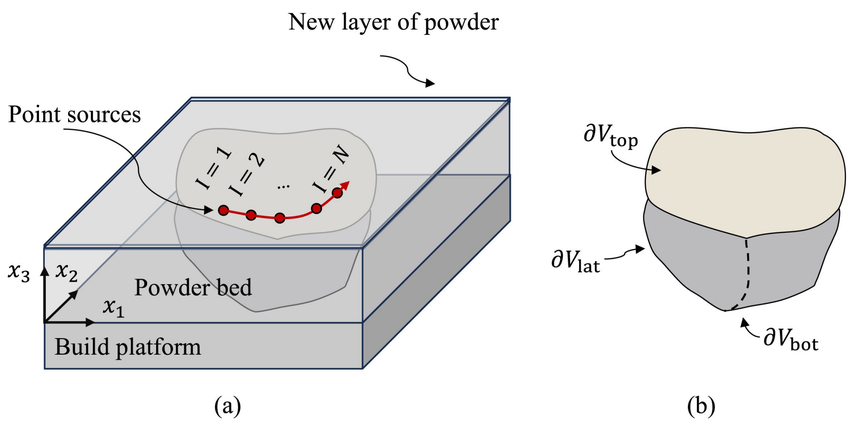
Laser Powder Bed Fusion (LPBF) generates steep, fast-moving thermal gradients on the order of the laser spot radius $r_{\text{laser}} \approx 20\,\mu\text{m}$. Conventional FEM requires either uniformly fine meshes or scan-wise adaptive remeshing — prohibitive at part scale. We present a **semi-analytical isogeometric framework** that:

- replaces the classical **image-source method** — which fails on realistic CAD — with an **IGA-discretised correction field**, enabling adiabatic BCs on arbitrary freeform geometries;
- delivers **80–258× CPU speed-up** over FEM at matched accuracy;
- is **heat-source agnostic**: the framework accommodates any analytical heat-source kernel admitting a half-space solution; demonstrated here with a Gaussian surface source.

LPBF Setup & Governing Equations



LPBF process: laser melts powder along a scan path.



Laser path discretised as N point sources; boundaries ∂V_{top}, ∂V_{lat}, ∂V_{bot}.

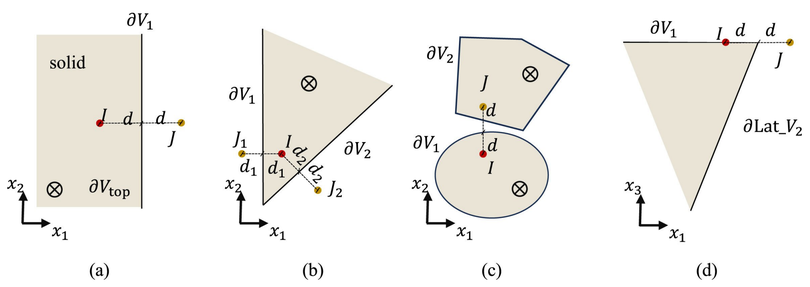
Heat equation. With density ρ, specific heat c_p, conductivity k, diffusivity $\alpha = k/(\rho c_p)$, and volumetric heat-source density Q, transient conduction in the build body V reads

$$\frac{\partial T}{\partial t} = \alpha\,\nabla^2 T + \frac{Q}{\rho c_p}, \quad \mathbf{x} \in V,\ t > 0, \qquad \frac{\partial T}{\partial \mathbf{n}} = 0 \text{ on } \partial V_{\text{lat}} \cup \partial V_{\text{top}}, \quad T = T_c \text{ on } \partial V_{\text{bot}}, \quad (1)$$

with initial condition $T(\mathbf{x}, 0) = T_c$ (radiative and convective losses neglected at the top surface). The laser scan is discretised as N instantaneous point sources fired at shifted times $\tau^{(I)} = t^{(I)} - r^2/(8\alpha)$, which **exactly reproduces** a Gaussian surface source of radius r in the half-space; arc-length spacing $v\,\Delta t$, with $\Delta t = 1 \times 10^{-5}\,\text{s}$.

Method: Analytical + Numerical Decomposition

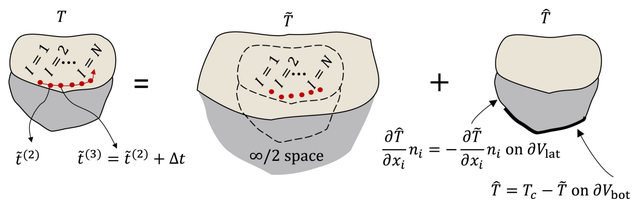
Image-source method fails for realistic geometries:



(a) Works for prismatic blocks; image sources fail at (b) corners, (c) disconnected solids, (d) varying cross-sections.

Our decomposition — split T into a **closed-form analytical part** $\widetilde{T}$ and a **smooth numerical correction** $\widehat{T}$:

$$\boxed{T = \widetilde{T} + \widehat{T}}$$



Analytical $\widetilde{T}$ — superposition of $K \le N$ instantaneous point-source Green's functions in the half-space (early sources with negligible contribution at time t are truncated):

$$\widetilde{T}(\mathbf{x}, t) = \sum_{I=1}^{K} \frac{E^{(I)}}{\rho c_p \left[4\pi\alpha(t - \tau^{(I)})\right]^{3/2}} \exp\left(-\frac{\|\mathbf{x} - \mathbf{x}^{(I)}\|^2}{4\alpha(t - \tau^{(I)})}\right) H\big(t - \tau^{(I)}\big),$$

with absorbed pulse energy $E^{(I)} = A\,P\,\Delta t$ (absorptivity A, laser power P) and shifted activation $\tau^{(I)} = t^{(I)} - r^2/(8\alpha)$.

Correction $\widehat{T}$ — homogeneous heat equation with non-homogeneous BCs that compensate the half-space residual on the true geometry:

$$\rho c_p\,\frac{\partial \widehat{T}}{\partial t} = k\,\nabla^2 \widehat{T}, \quad \frac{\partial \widehat{T}}{\partial \mathbf{n}} = -\frac{\partial \widetilde{T}}{\partial \mathbf{n}} \text{ on } \partial V_{\text{lat}} \cup \partial V_{\text{top}}, \quad \widehat{T} = T_c - \widetilde{T} \text{ on } \partial V_{\text{bot}}.$$

Conclusions & References

- IGA correction $\widehat{T}$ enforces adiabatic BCs on realistic CAD geometries.
- C^{p-1} cubic NURBS retains $\le 10\%$ error at elements 5–13 × r_{laser}.
- **80**× vs Abaqus C3D4, **258**× vs Abaqus C3D20 at matched accuracy.

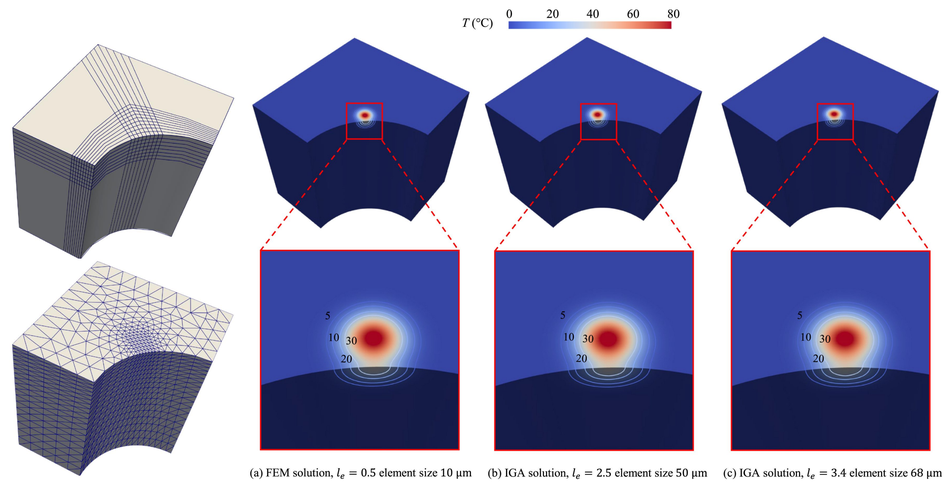
[1] Yang, Y., Ji, Y., Möller, M., & Ayas, C. (2026). Efficient thermal simulation in metal additive manufacturing via semi-analytical isogeometric analysis. *Computer Methods in Applied Mechanics and Engineering*, 457, 118992.

[2] Yang, Y., Ji, Y., Möller, M., & Ayas, C. (2025). Computational efficient process simulation of geometrically complex parts in metal additive manufacturing. *International Journal of Heat and Mass Transfer*, 248, 127059.

Convergence: Curved Boundary

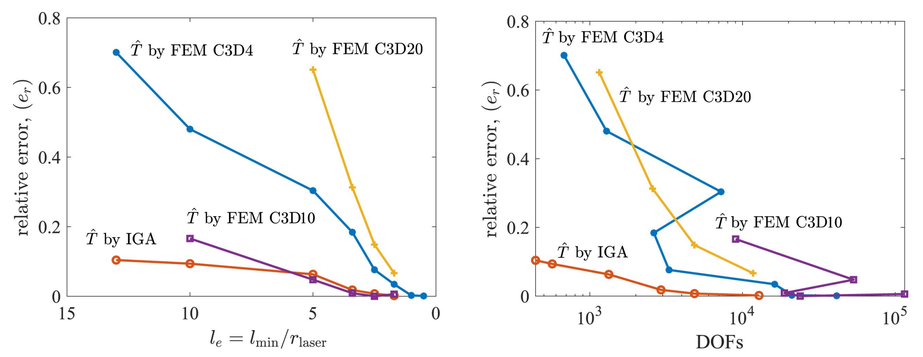
Setup. 2 mm-edge cube with a quarter-cylinder ($R_c = 1\,\text{mm}$) removed; point source 100 µm from the curved boundary. Ti-6Al-4V, $P = 82.5\,\text{W}$, $A = 0.77$, $r_{\text{laser}} = 20\,\mu\text{m}$. Mesh fineness reported as $l_e = l_{\min}/r_{\text{laser}}$ (smaller = finer).

Temperature contours at matched coarse mesh $l_e = 3.4$: IGA reproduces the FEM reference ($l_e = 0.5$).



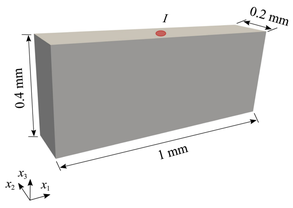
(a) FEM solution, l_e = 0.5 element size 10 µm (b) IGA solution, l_e = 2.5 element size 50 µm (c) IGA solution, l_e = 3.4 element size 68 µm

Convergence of the total field T in relative L^2-error vs (a) element size l_e, (b) DOFs:



- **IGA retains $e_r \lesssim 10\%$ even at $l_e = 13$** — elements 13× the laser radius;
- at matched DOFs: **1–2 orders of magnitude lower error** than FEM C3D4 / C3D20.

Thin-Wall: CPU Speed-up



1 mm×0.2 mm×0.4 mm thin wall.

Uniform mesh, target $e_r \le 5\%$; FEM in Abaqus, IGA in G+Smo; Intel i7-11700 @ 2.5 GHz.

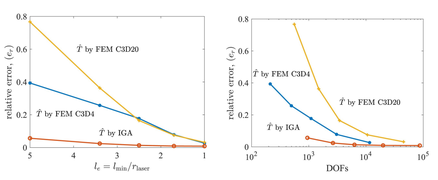
Method	$l_{\min}$ [µm]	DOFs	CPU/step [s]
IGA (quad.)	68	2639	**0.132**
Abaqus C3D4	20	11 536	10.625
Abaqus C3D20	20	45 261	34.043

Speed-up at matched 5% error:

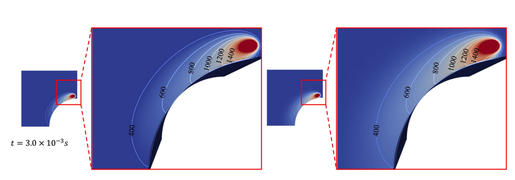
$$\frac{T_{\text{Abaqus, C3D4}}}{T_{\text{IGA}}} = \mathbf{80}\times, \qquad \frac{T_{\text{Abaqus, C3D20}}}{T_{\text{IGA}}} = \mathbf{258}\times.$$

C^{p-1}-continuous NURBS represents through-thickness gradients smoothly across multiple elements, whereas C^0 FEM must reduce $l_{\min}$ to resolve the same gradient.

Contour Scan & Butterfly Part



Convergence of $\widehat{T}$ vs (a) l_e, (b) DOFs.



Temperature contours; FEM ref ($l_e = 0.5$) vs IGA ($l_e = 5$).

Contour scan along curved boundary. At $t = 3 \times 10^{-3}$ s, the heat-flux residual (BC-violation measure on ∂V_{lat}): FEM 17.30% vs IGA's 8.03% on a mesh 10× **coarser** (5 415 vs 45 773 DOFs).

Butterfly part (cubic-NURBS volumetric parametrisation, IGA $l_e = 5$, **27 456 DOFs**) — adiabatic BCs on a free-form CAD surface, where the classical image-source method is **inapplicable**: