Analysis-suitable parameterization for isogeometric analysis

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Outline

1. Background and motivation
2. Overview of the algorithms
3. Applications
4. Conclusions and outlook

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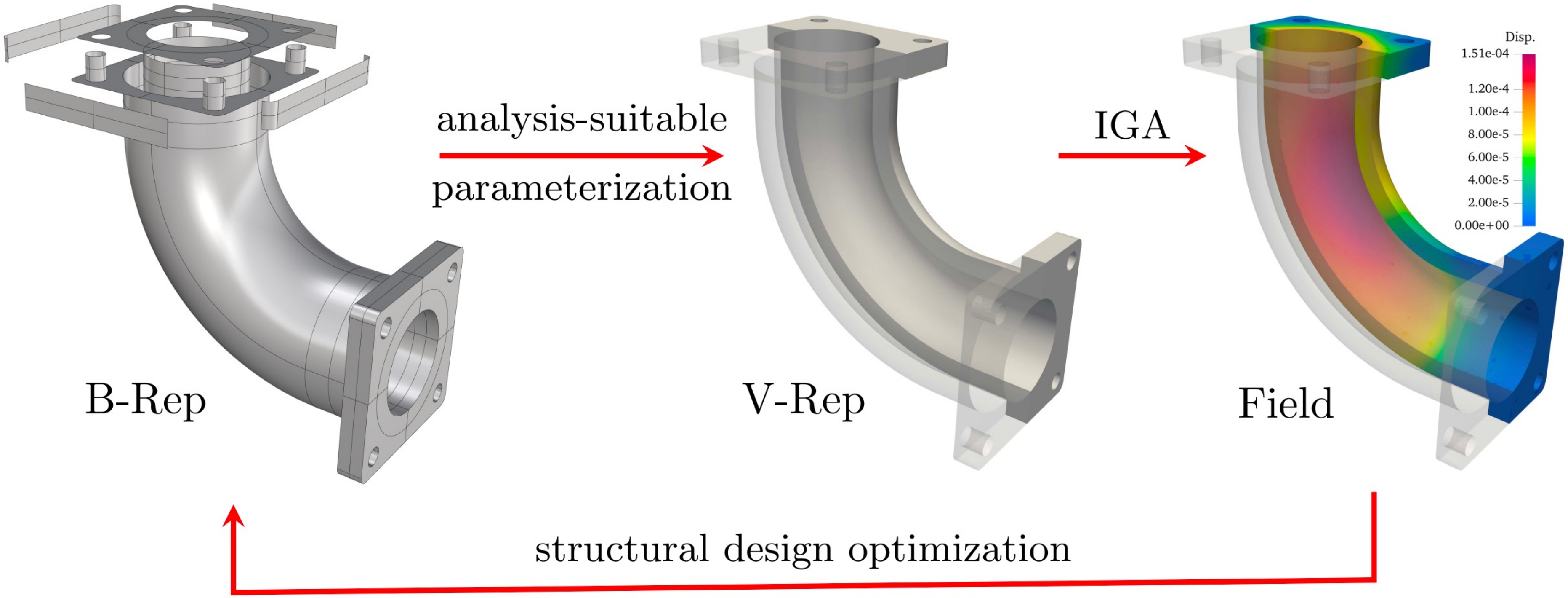
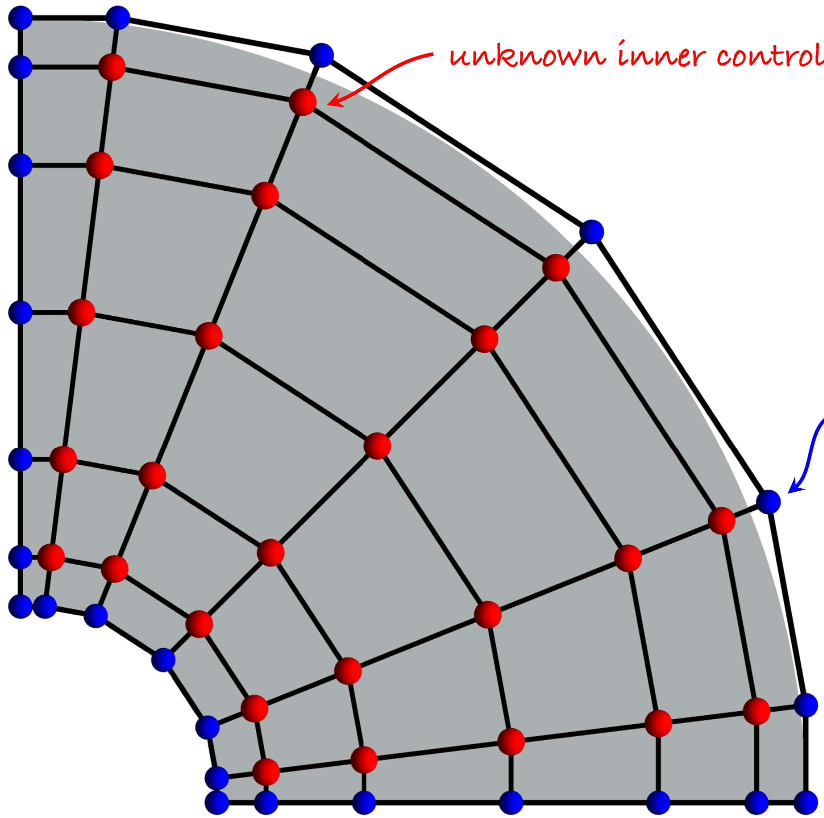
Domain parameterization

- CAD models are usually represented by **boundary representation** (B-Rep);
- However, IGA requires an internal **spline-based parameterization** (V-Rep).



Problem statement: Domain parameterization



$$\boldsymbol{x}(\boldsymbol{\xi}) = \underbrace{\sum_{i\in\mathcal{I}_I} \mathbf{P}_i R_i(\boldsymbol{\xi})}_{\text{unknown}} + \underbrace{\sum_{j\in\mathcal{I}_B} \mathbf{P}_j R_j(\boldsymbol{\xi})}_{\text{known}}.$$

GOAL: to construct **unknown inner control points $\mathbf{P}_i$** (or basis functions R_i) such that x ensures **bijectivity** and exhibits **optimal orthogonality** and **uniformity**.

➢ **Parameterization quality** significantly affects **downstream analysis!**

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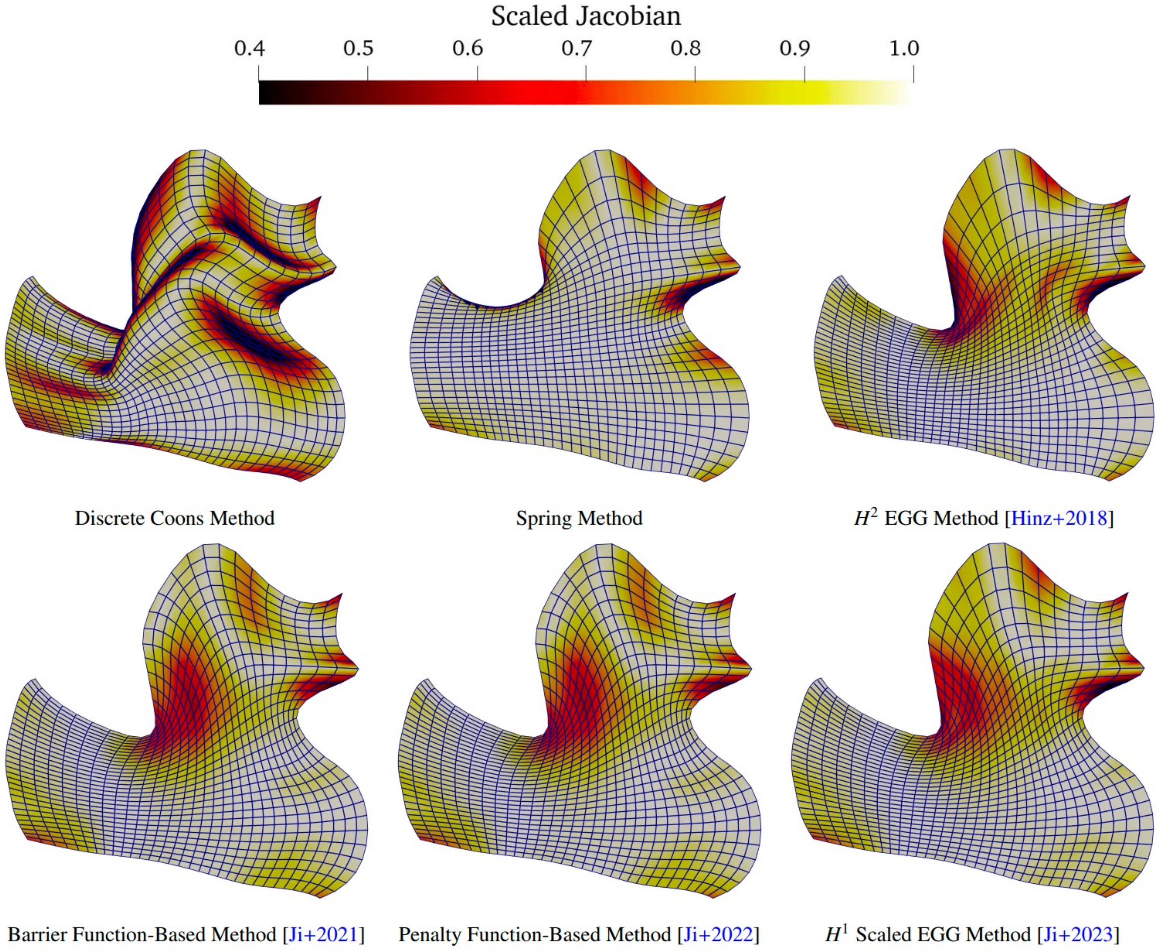
Available methods in G+Smo



- Algebraic methods:
  - Coons patch
  - Spring patch
- Optimization-based methods:
  - Barrier-function-based [1]
  - Penalty-function-based [2]
- PDE-based methods:
  - Elliptic grid generation [3]
  - Improved EGG [4]

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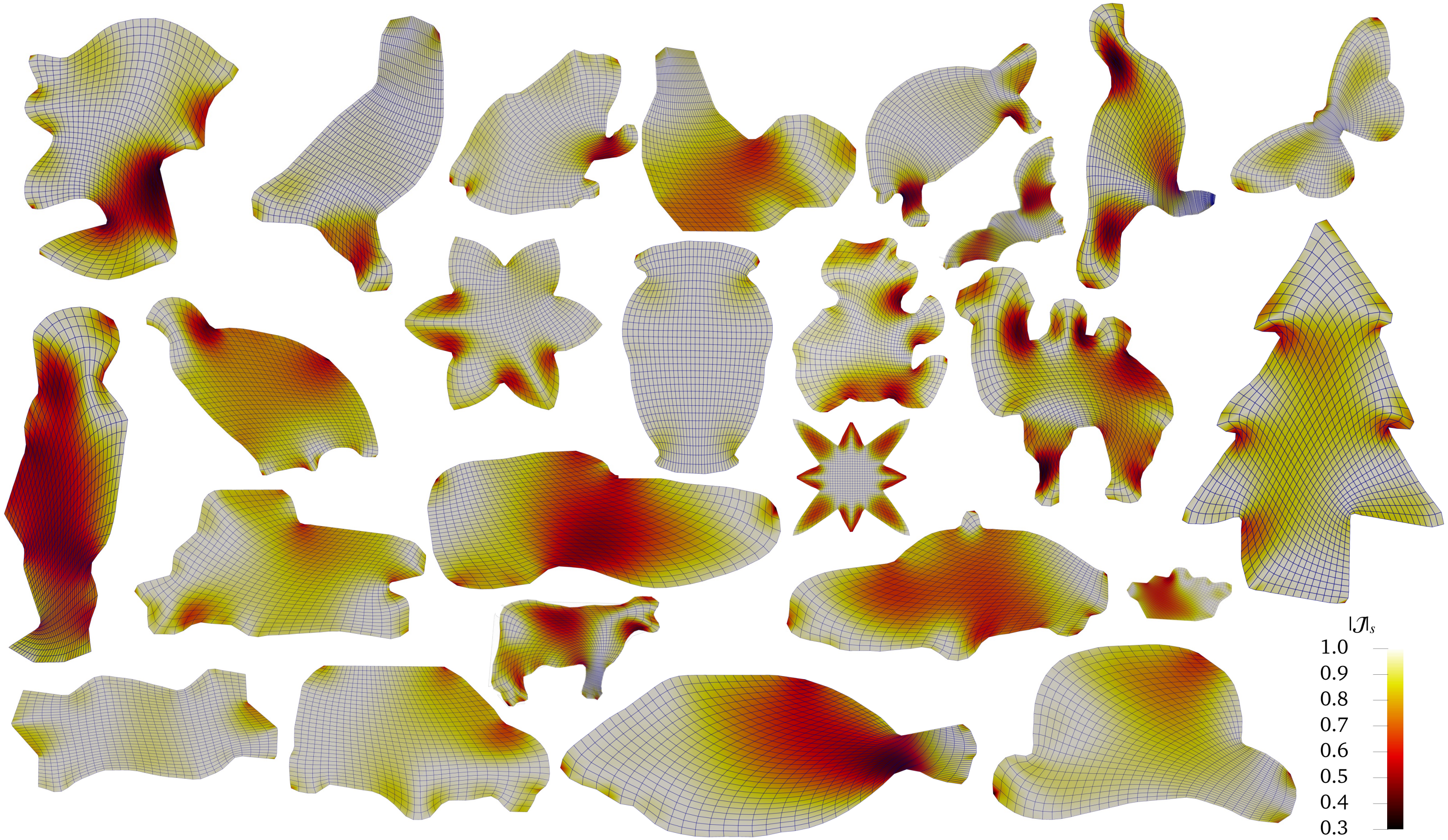
1. Ji, Y. et al. (2021). JCAM, *396*, 113615.
2. Ji, Y. et al. (2022). CAGD, 94, 102081.
3. Hinz, J. et al. (2018). CAGD, *65*, 48-75.
4. Ji, Y. et al. (2023). *CAGD*, *102*, 102191.

Planar Parameterization Test Dataset (977 models)

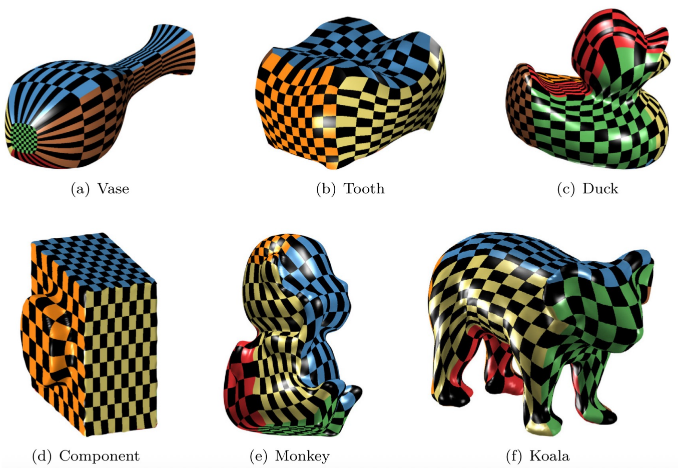


Success rates
PDE (Hinz, J. et al. (2018)): 62.23%
PDE (Ji et al. 2023): 73.80%
Barrier (Ji et al. 2021): **98.36%**
Penalty (Ji et al. 2021): 97.85%

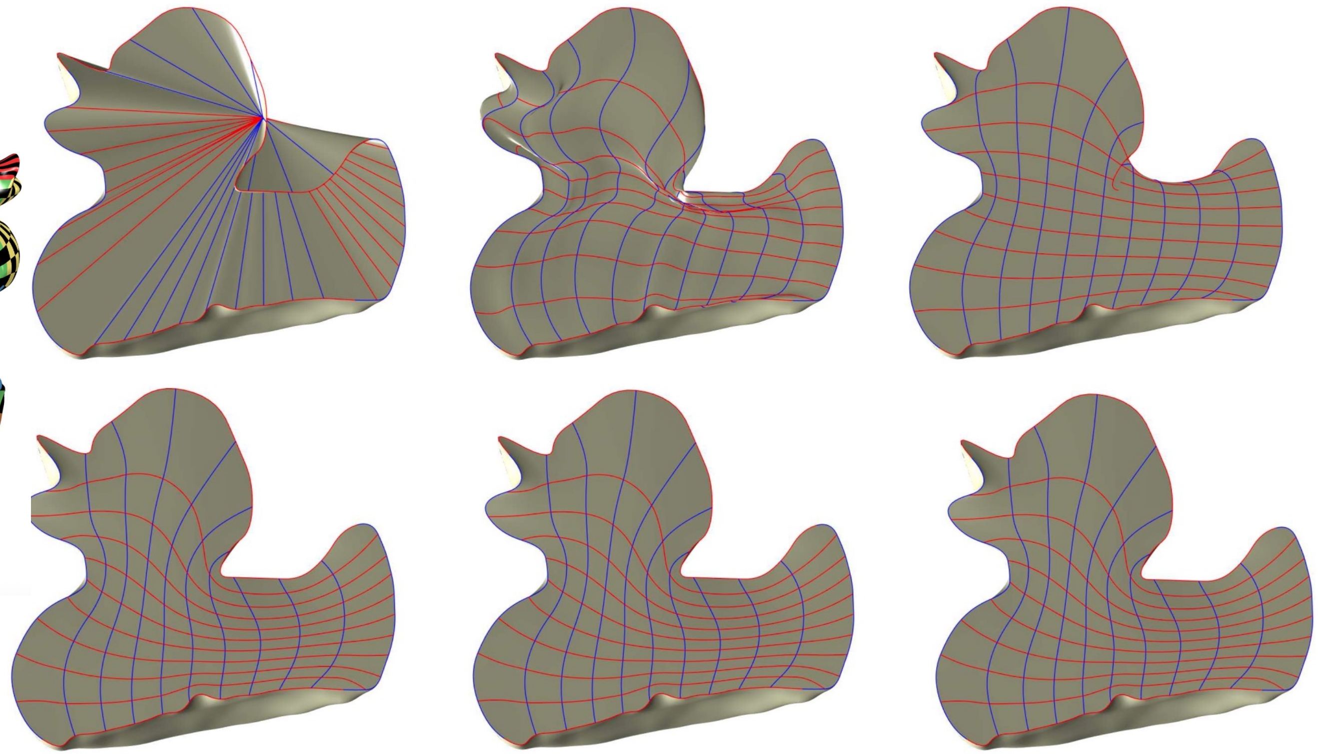
Computational time:
PDE-based: ~0.2 sec
opt-based: ~2 sec

Ji, Y., Möller, M., Verhelst, H.M. (2024). Design Through Analysis. In: Bodnár, T., Galdi, G.P., Nečasová, Š. (eds) Fluids Under Control. Advances in Mathematical Fluid Mechanics. Birkhäuser, Cham.

Strong similarity from different initializations

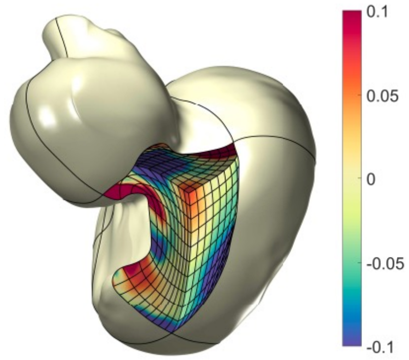


Volumetric modes used for testing

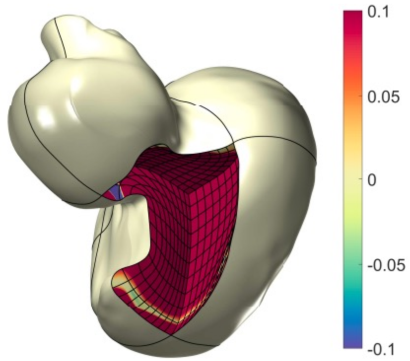


Same point   Coons Patch   Smoothness energy

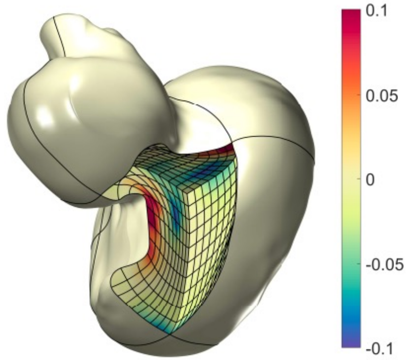
Comparisons with existing methods



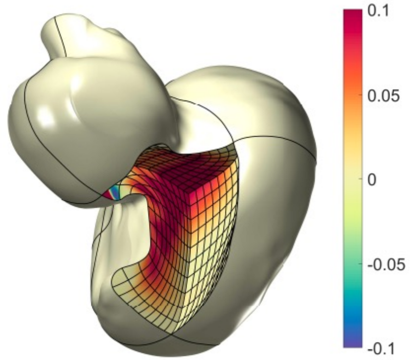
$m_{SJ}^{Ours} - m_{SJ}^{Pan}$



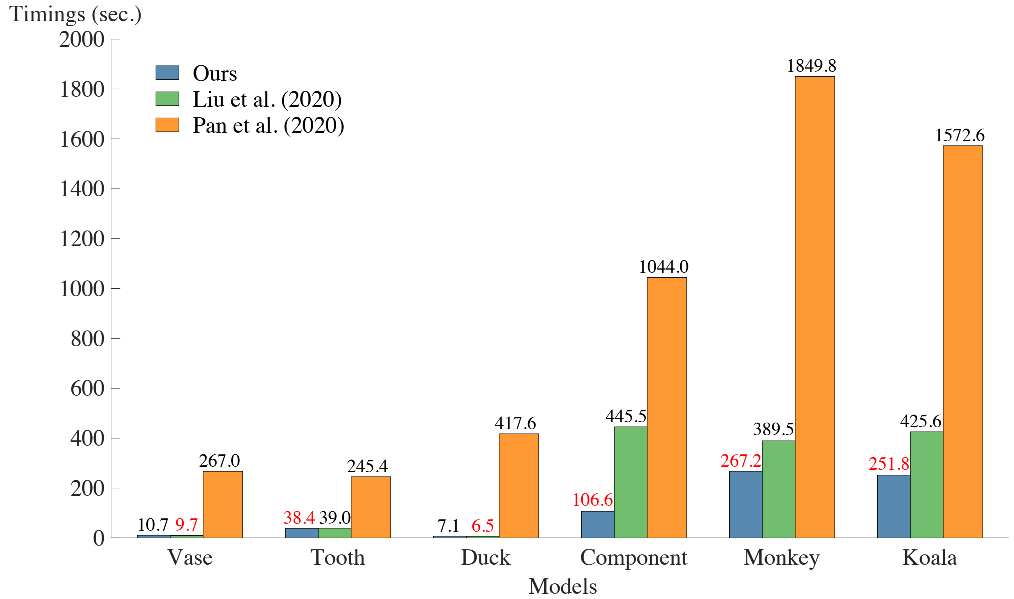
$m_{unif.}^{Pan} - m_{unif.}^{Ours}$



$m_{SJ}^{Ours} - m_{SJ}^{Liu}$



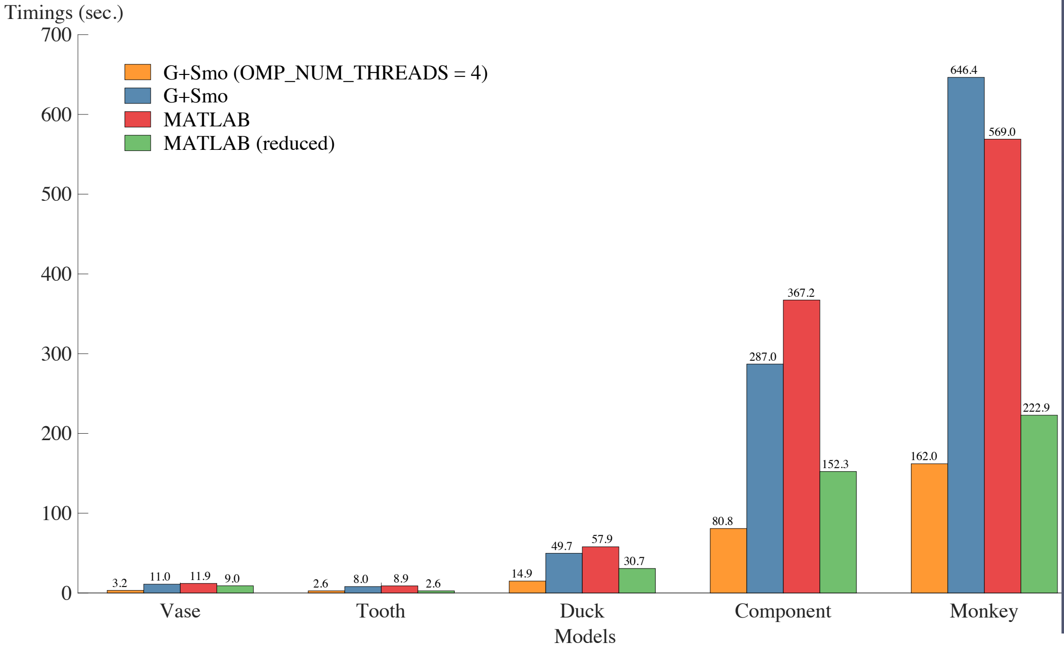
$m_{unif.}^{Liu} - m_{unif.}^{Ours}$

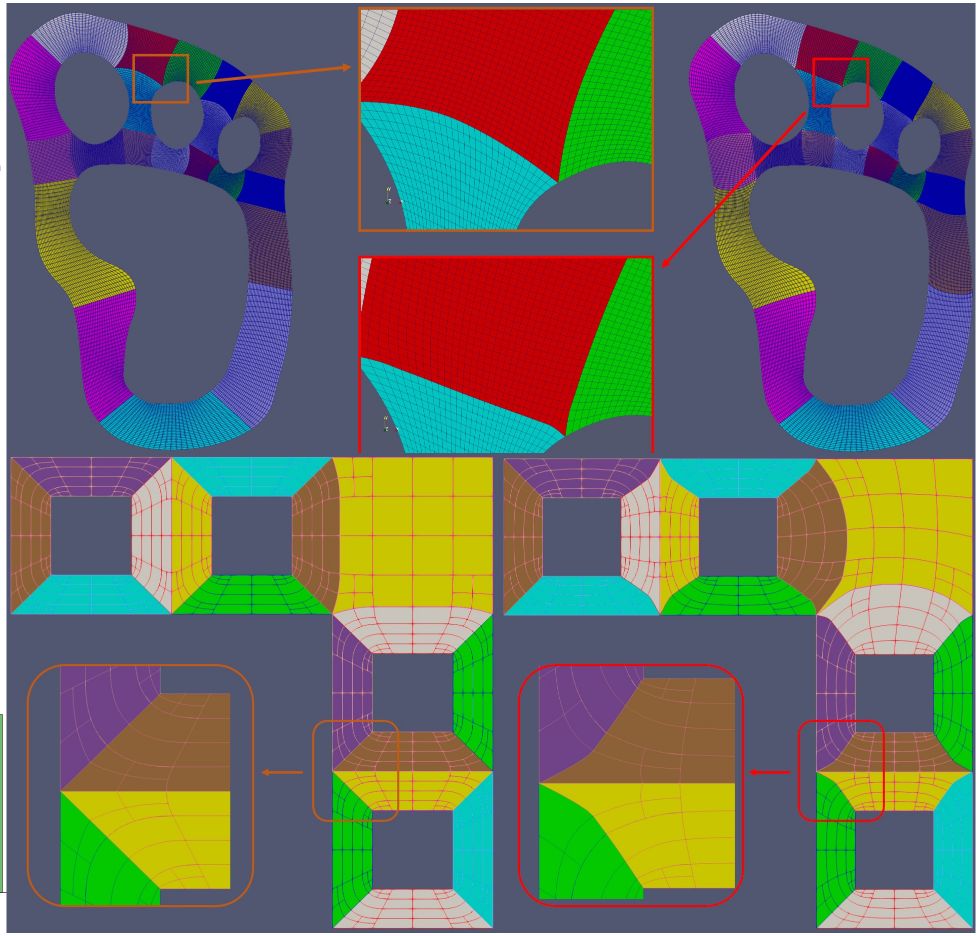


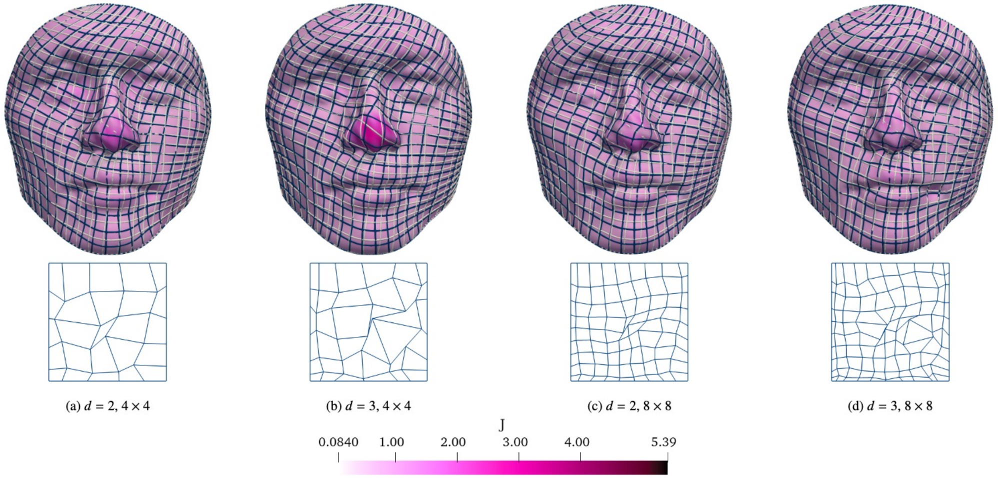
- **Positive values** indicate our method performs better;
- Efficiency comparison (MATLAB vs. C++ (Pan+ & Liu+):
  - Significantly **faster** than Pan et al. (2020);
  - Large-Scale Models: Outperforms Liu et al. (2020).

1. Pan, M., Chen, F., & Tong, W. (2020). CMAME, 359, 112769.
2. Liu, H., Yang, Y., Liu, Y., & Fu, X. M. (2020). CAGD, 79, 101853.

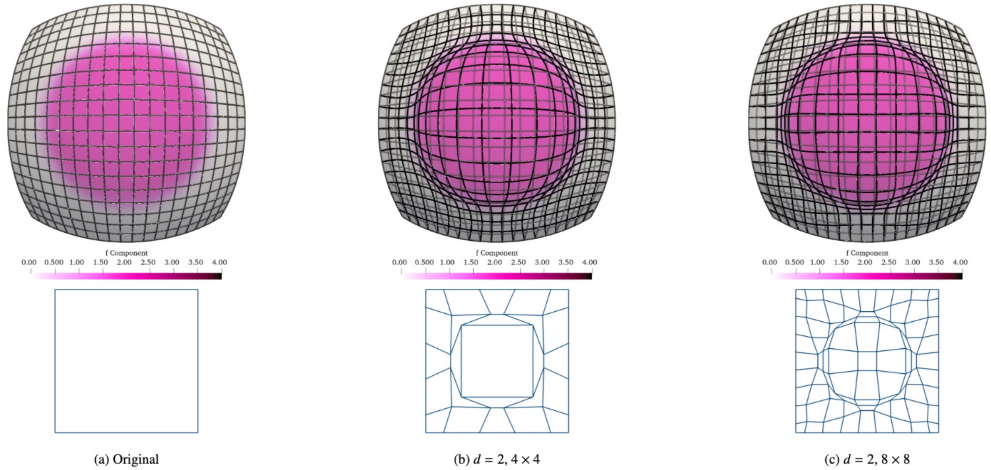
- In our released G+Smo implementation, **3-4x speed-up using OpenMP (!)**;
- Suitable for **multi-patch** and **THB-spline parameterization**;







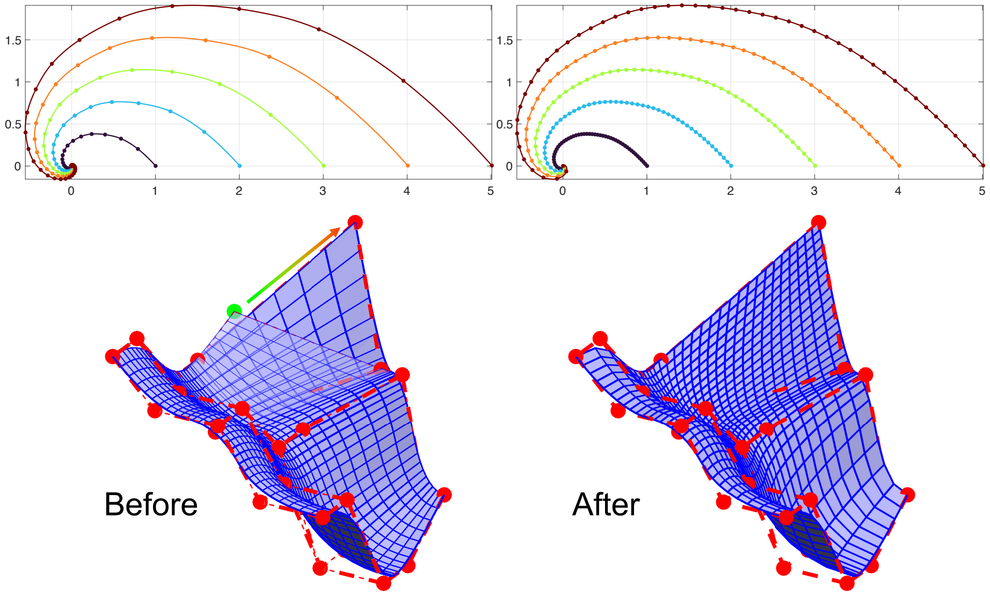
(a) $d = 2, 4 \times 4$ (b) $d = 3, 4 \times 4$ (c) $d = 2, 8 \times 8$ (d) $d = 3, 8 \times 8$

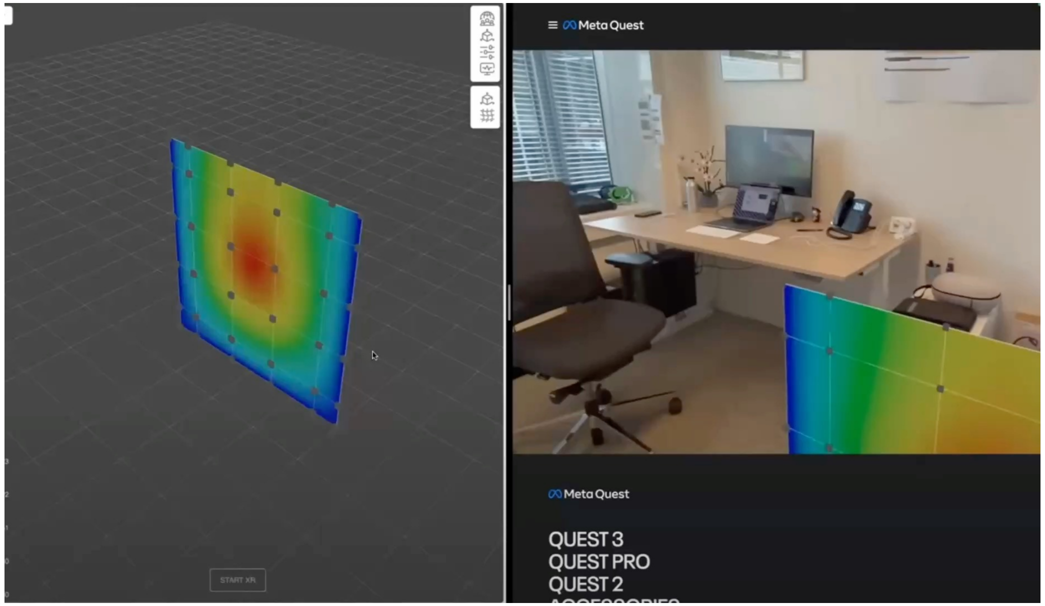


(a) Original (b) $d = 2, 4 \times 4$ (c) $d = 2, 8 \times 8$

- **A composite spline relocation** framework for surface-based IGA, integrating **harmonic map-based mesh redistribution** with spline-based techniques;

Composite spline relocation for isogeometric analysis on surfaces
joint work with Carlotta Giannelli, Sofia Imperatore, & Hugo Verhelst

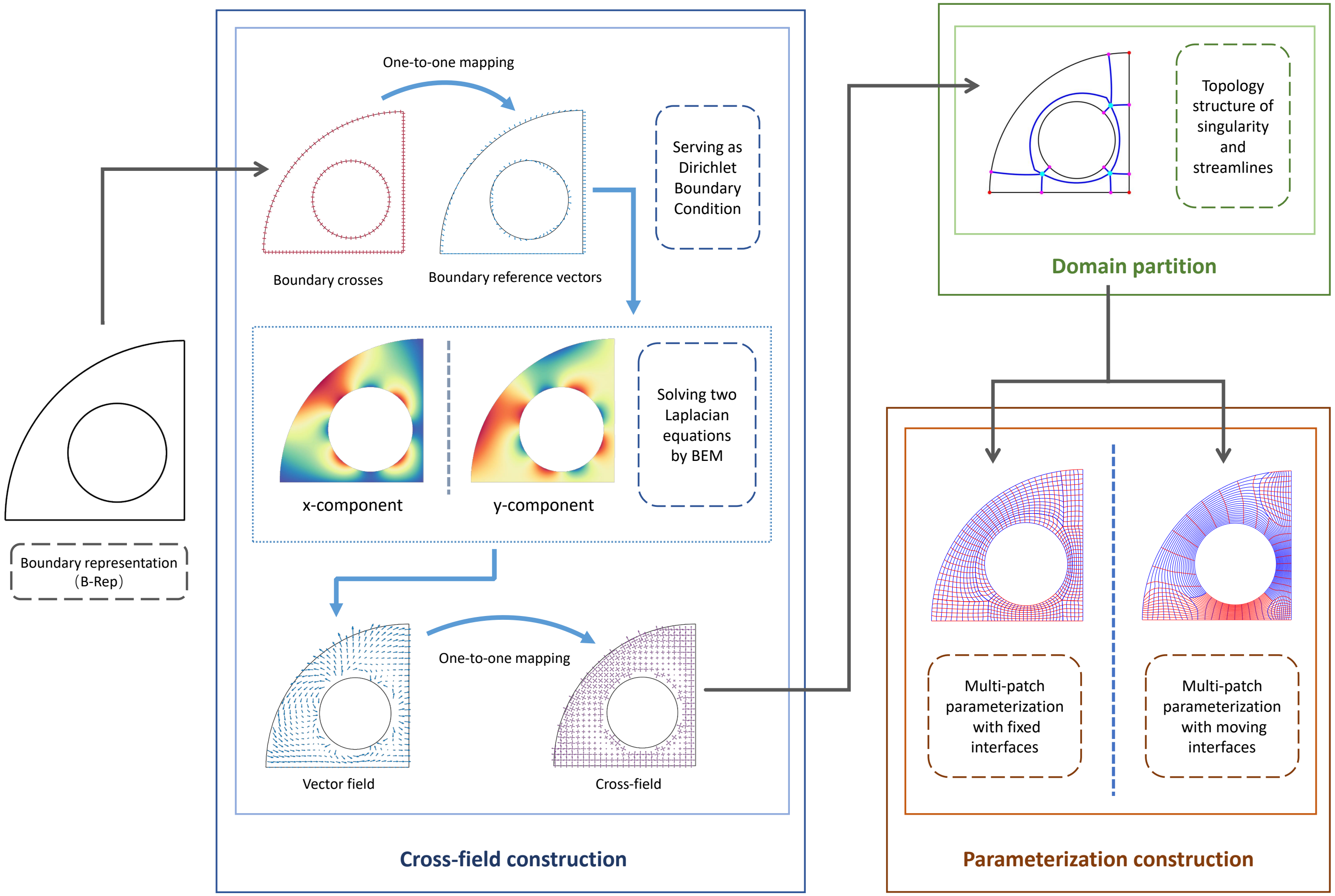


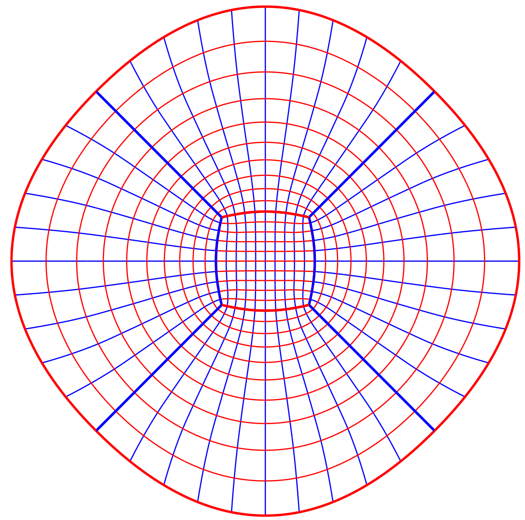


- Curve/surface reparameterization while keeping the geometry;

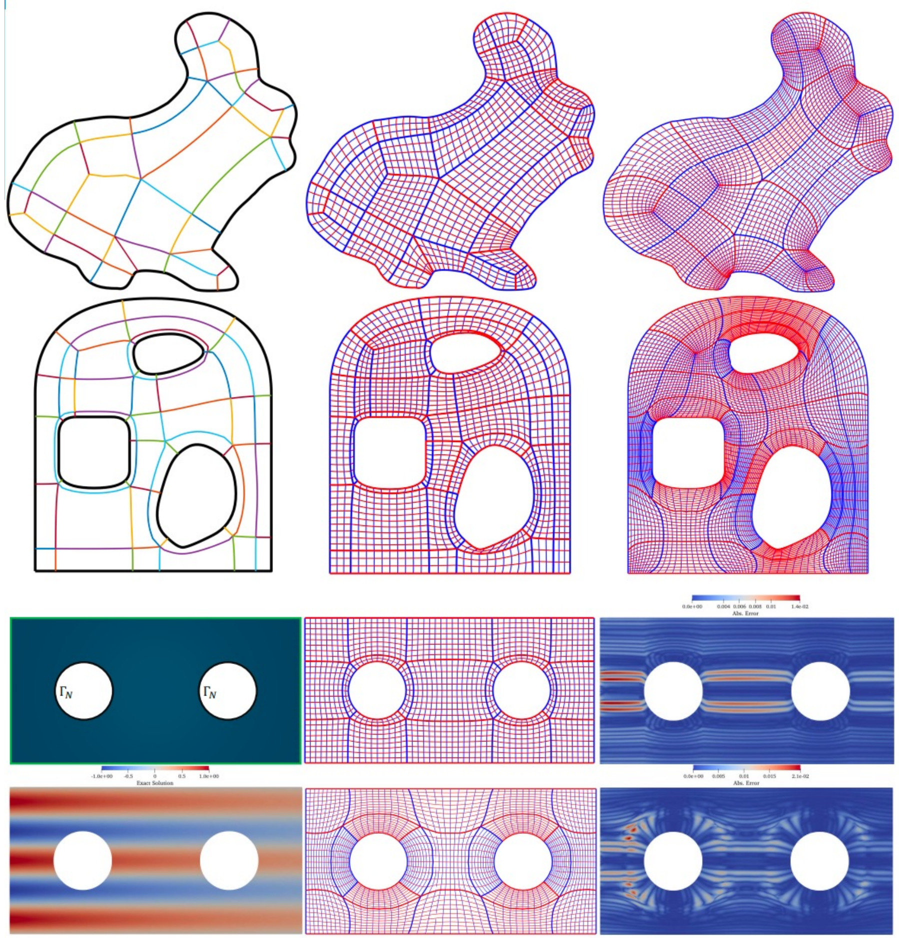
Design Through Analysis (DTA) Tools - Matthias
Wed. 10:45-12:00 Software session #2

Multi-patch parameterization using cross-field

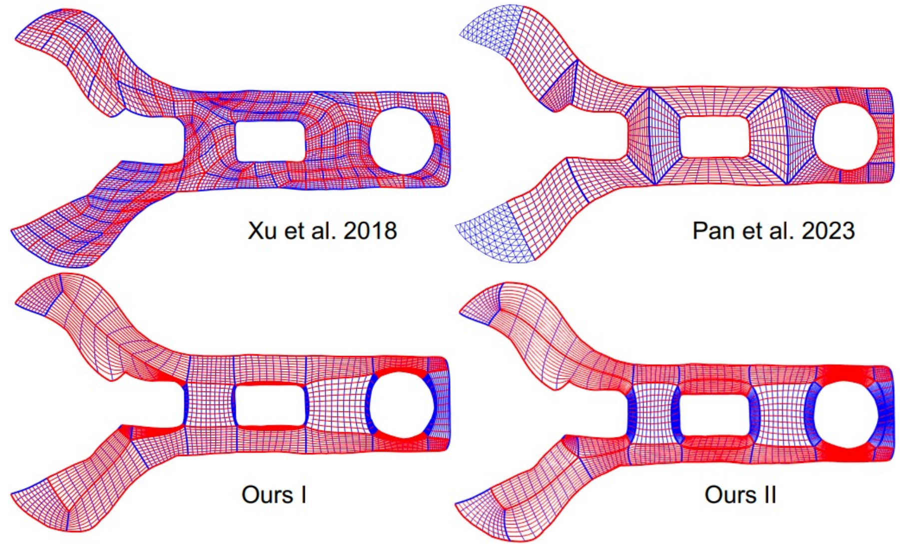




Zhang, Y., Ji, Y., & Zhu, C. G. (2024). Multi-patch parameterization method for isogeometric analysis using singular structure of cross-field. *Computers & Mathematics with Applications*, *162*, 61-78.

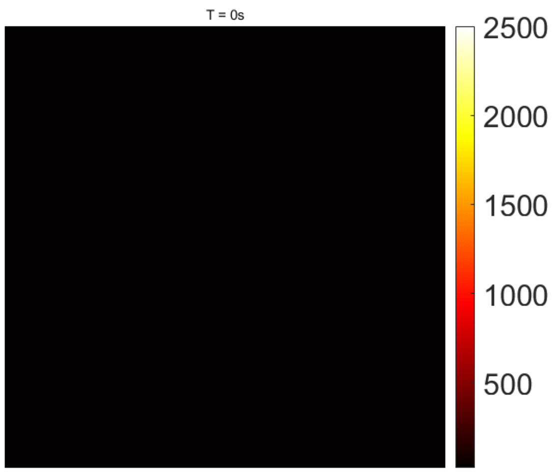


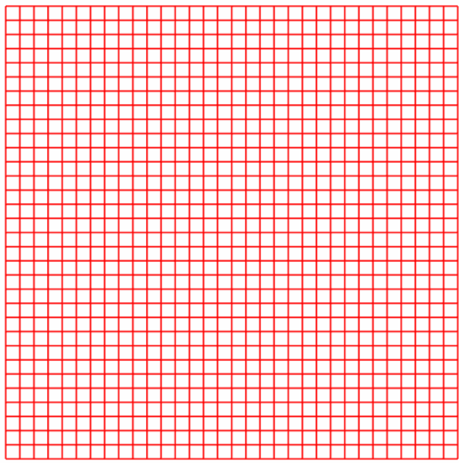
Model	#Patch	Method	$\|\mathcal{J}\|_s$ min.	$\|\mathcal{J}\|_s$ avg.	*unif.* min.	*unif.* avg.	*unif.* max.
rabbit	33	Coons	-0.8593	0.9628	0.7030	**0.9410**	1.0524
		fixed-I	**0.2204**	**0.9504**	0.6103	0.9544	**0.9982**
		moving-I	0.02918	0.9283	**0.0000**	0.9550	1.0000
3 holes	46	Coons	-0.5492	0.9710	0.8008	0.9573	1.0958
		fixed-I	**0.1545**	**0.9716**	0.8007	0.9573	0.9978
		moving-I	0.1461	0.9361	**0.6791**	**0.9571**	**0.9968**

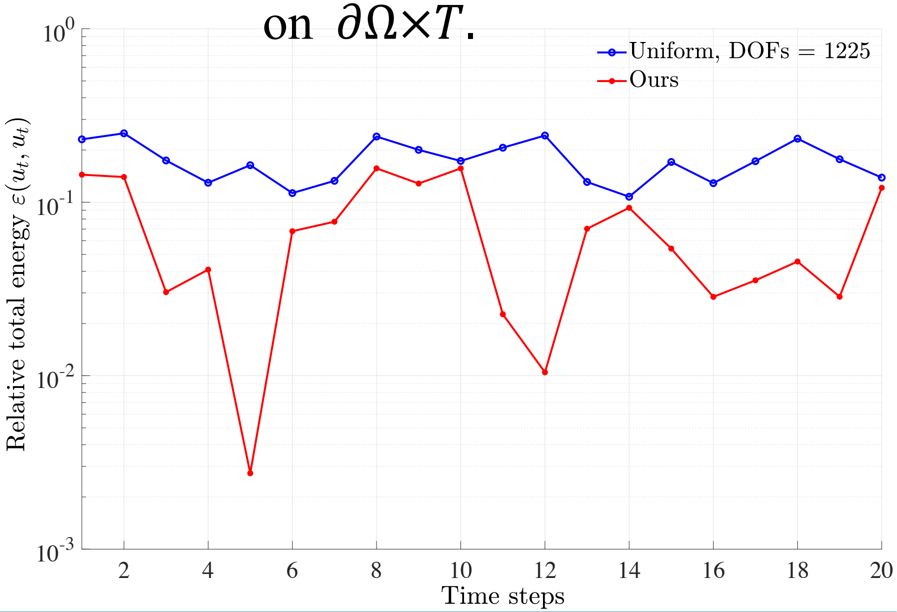


- Consider a two-dimensional linear heat transfer problem with a moving Gaussian heat source:

$$\begin{cases} C_p\rho\dfrac{\partial u(\boldsymbol{x},t)}{\partial t} - \nabla\cdot\big(\kappa\nabla u(\boldsymbol{x},t)\big) = f(\boldsymbol{x},t), & \text{in } \Omega\times T, \\ u(\boldsymbol{x},t) = u_0, & \text{in } \Omega, \\ \kappa\nabla u(\boldsymbol{x},t) = 0, & \text{on } \partial\Omega\times T. \end{cases}$$

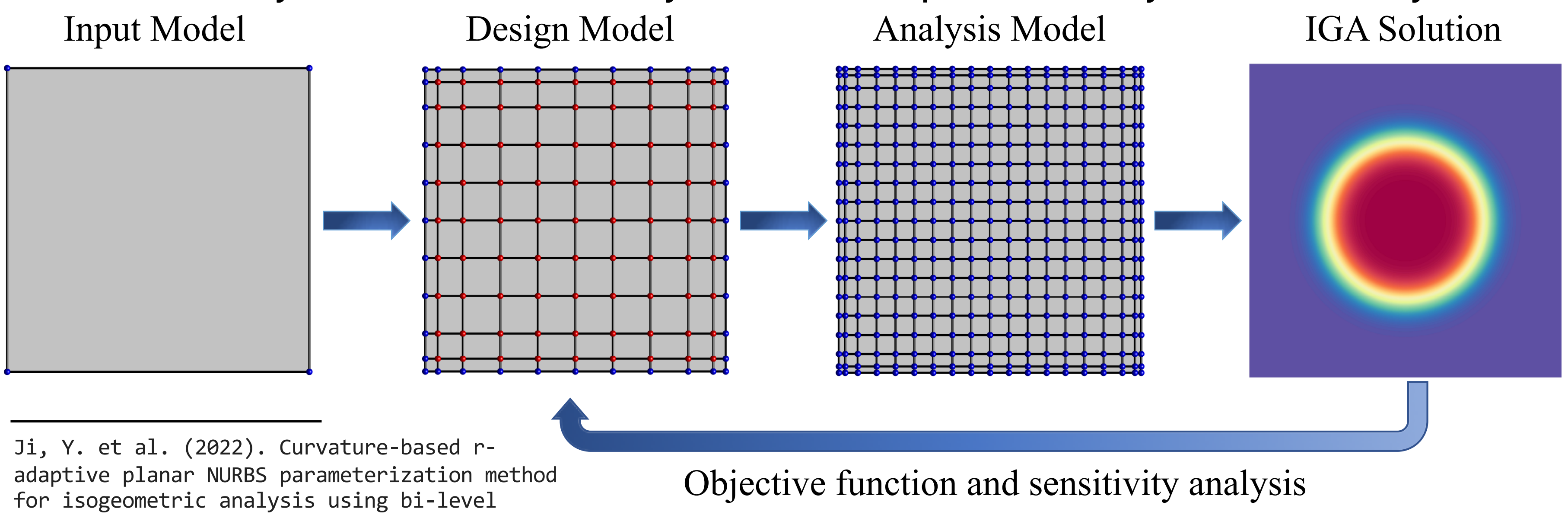






Anisotropic parameterizations are often solution-dependent:

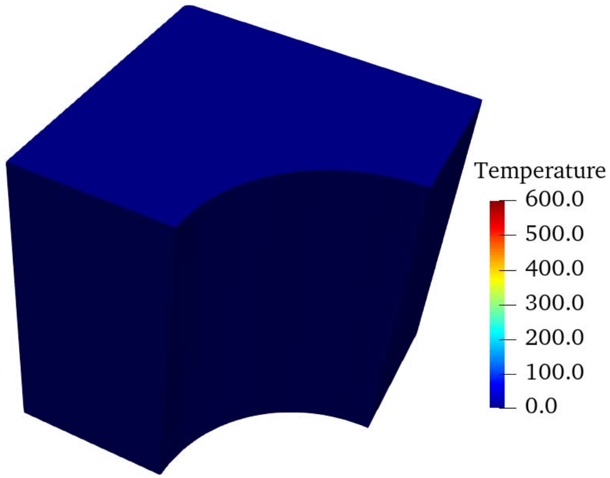
- Need **good numerical solution accuracy** to drive parameterization;
- Adjust as few control points as possible **for high efficiency**;
- **Bi-level strategy:** a coarse level (design model) to update the parameterization for efficiency and a fine level (analysis model) to perform analysis for accuracy.



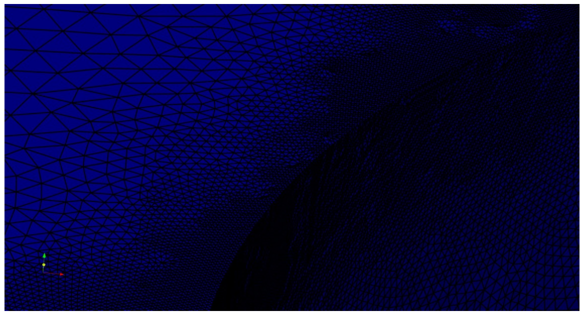
Ji, Y. et al. (2022). Curvature-based r-adaptive planar NURBS parameterization method for isogeometric analysis using bi-level approach. *Computer-Aided Design, 150,* 103305.

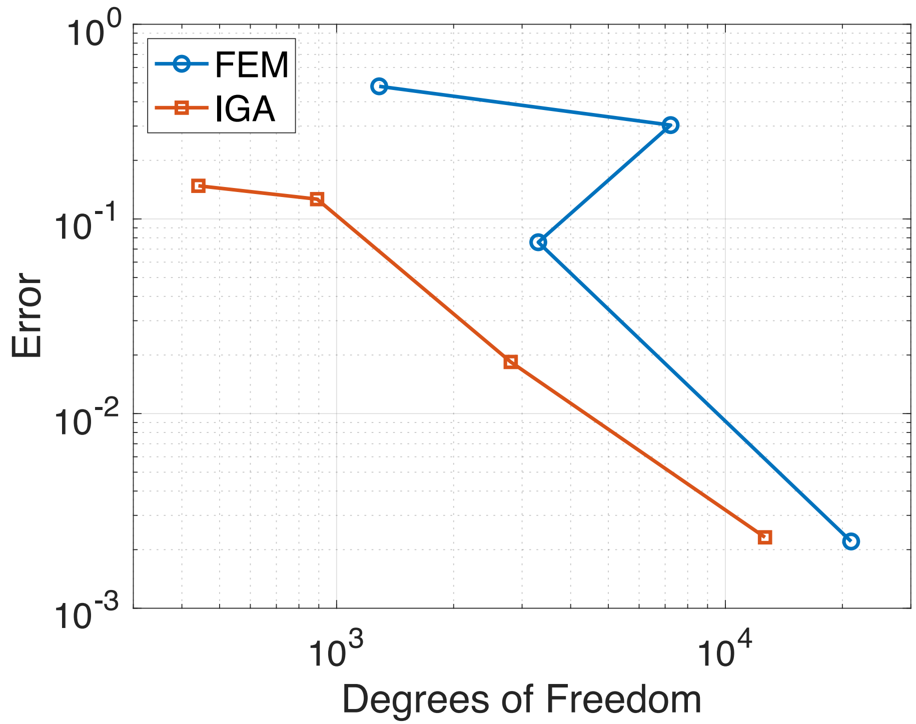
Semi-analytical heat transfer in metal additive manufacturing

IGA + Gismo



FEM + Abaqus





1. Yang, Y., Ji, Y., Möller, M., & Ayas, C. (2025). Computational efficient process simulation of geometrically complex parts in metal additive manufacturing. International Journal of Heat and Mass Transfer, 248, 127059.
2. Semi-analytical heat transfer modeling in metal additive manufacturing using isogeometric analysis (manuscript in preparation)

Physics-informed neural parameterization for IGA

Network & Training

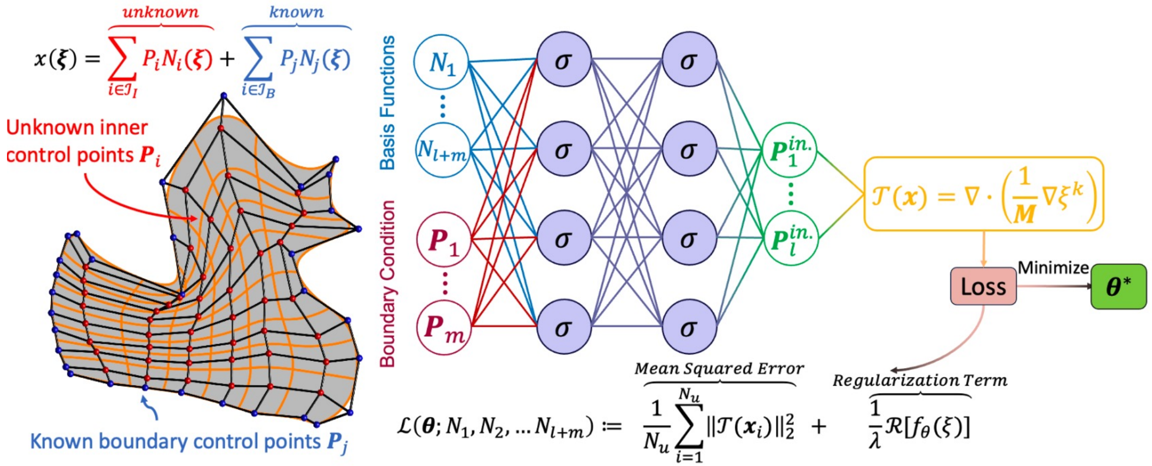
- 4 fully-connected layers, tanh activations
- Optimizer: Adam, learning rate = 1e−3
- Epochs: 50,000

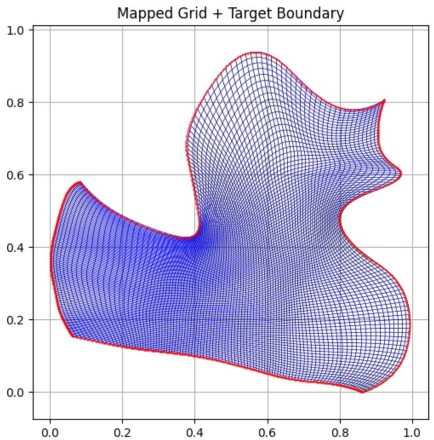
Loss Function

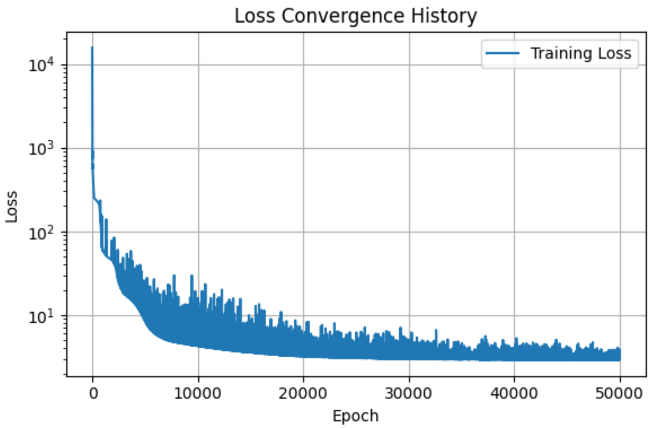
- Interior Loss: Jacobian-regularized Winslow energy ⇒ smooth & invertible
- Boundary Loss: MSE between predicted and B-spline-defined boundaries

✅Powerful auto differentiation tools make it easy to implement;
❌No better than the current ”classical” method; **12 min 1.4 sec. (PINN) vs 0.014 sec. (PDE) vs 0.0673 sec. (opt.)**

Not an “AI guy” (yet) myself, but very interested in bridging AI with geometry and simulation.







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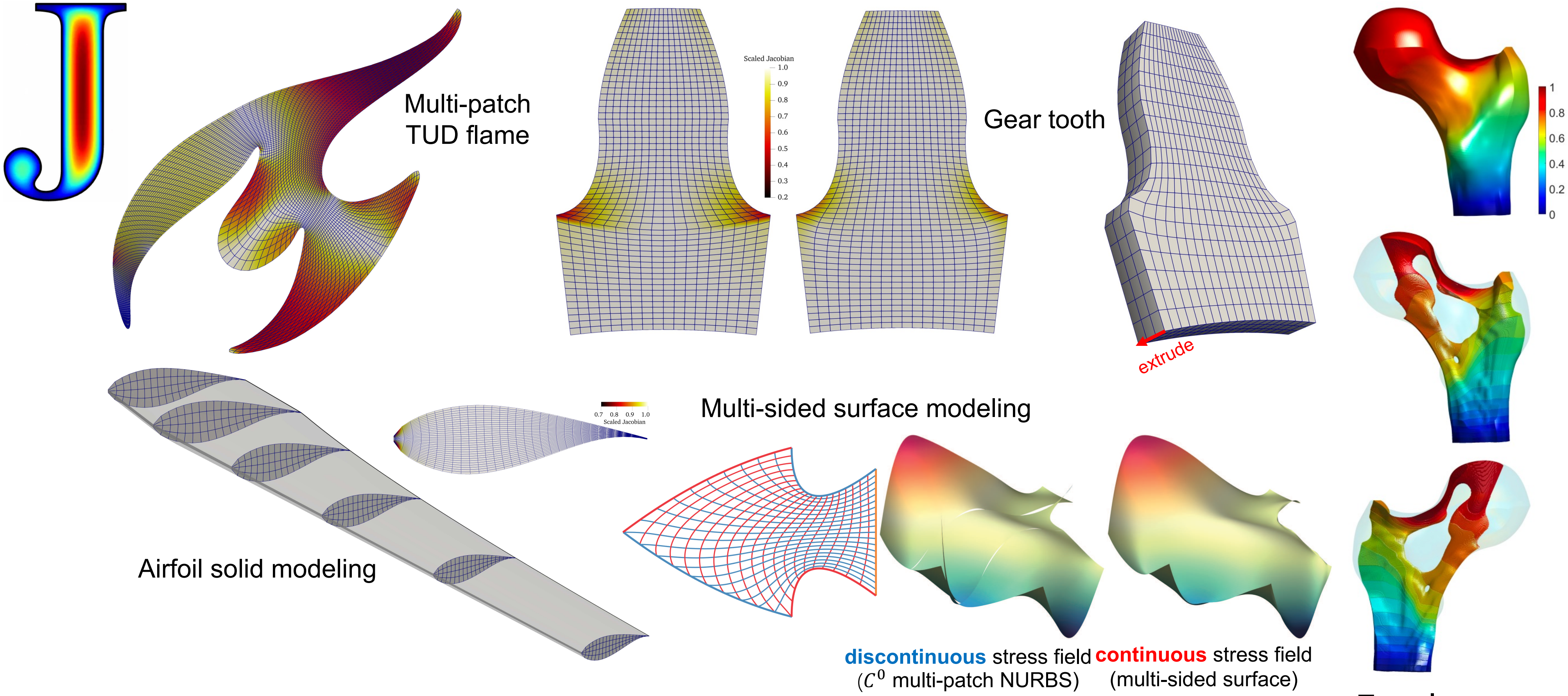
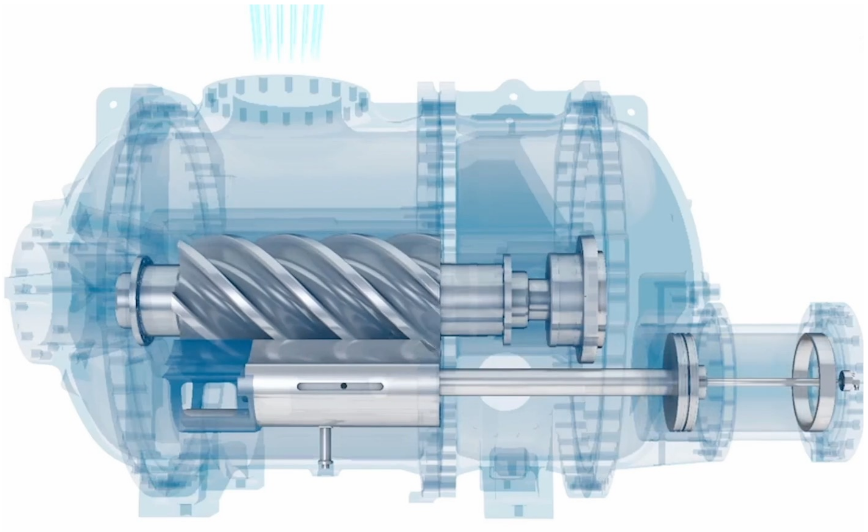
Applications: Academic use cases

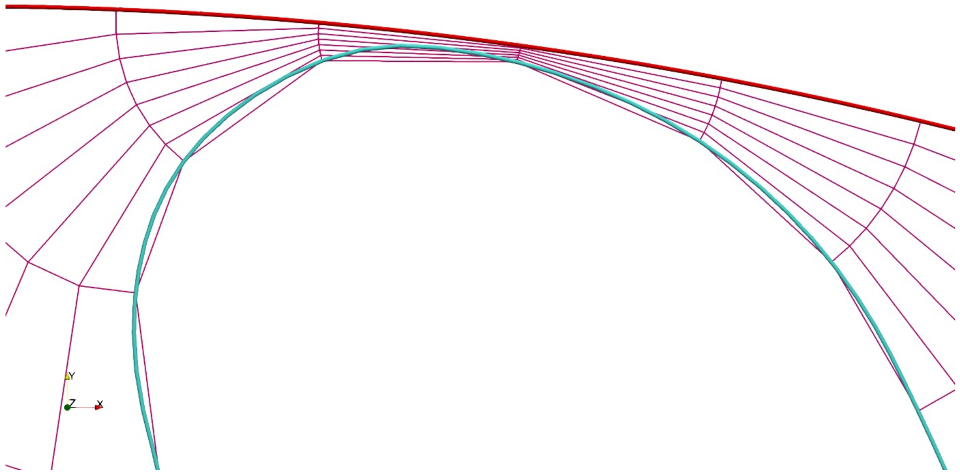


Applications: Industrial use case

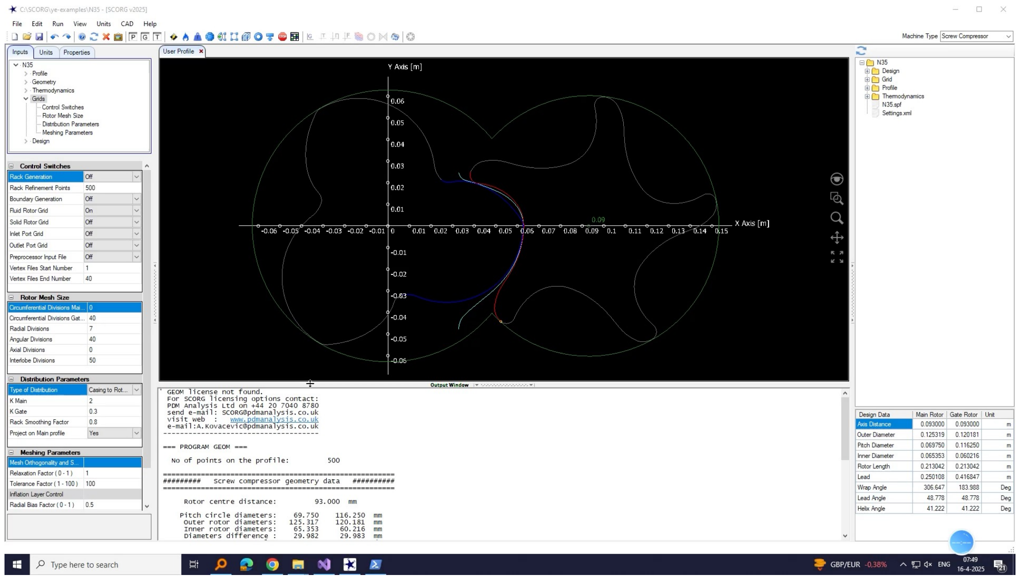


Rotary twin-screw compressor

- **Structured mesh generation** is a crucial preprocessing step in the simulation-based analysis of twin-screw machines.
- However, the existing mesh generators typically produce only **linear meshes with straight-sided cells**;
- **Analysis-suitable**, **high-order NURBS parameterizations.**

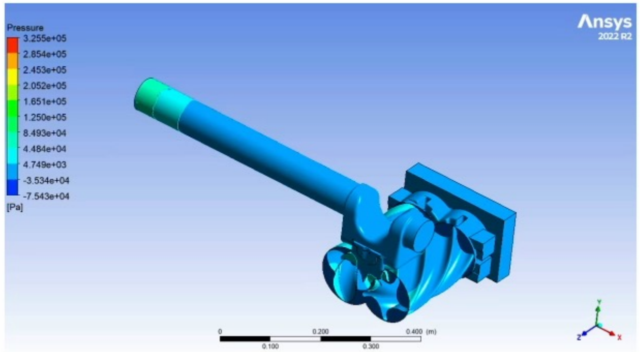


Source: https://www.gascompressors.co.uk/technologies/oil-floodedscrew-compressor/



Scorg

SplineMesh



ANSYS CFX simulation

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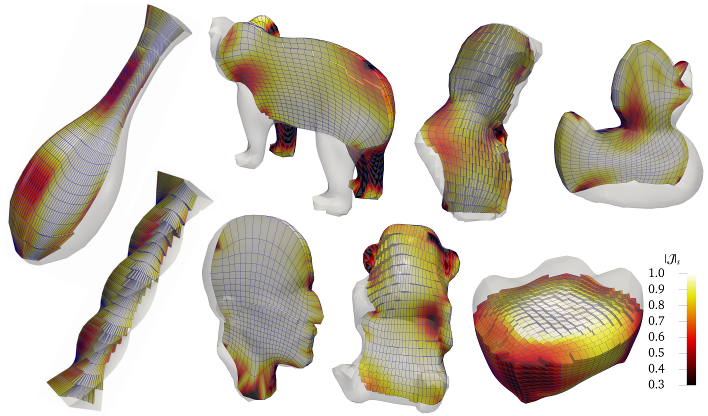
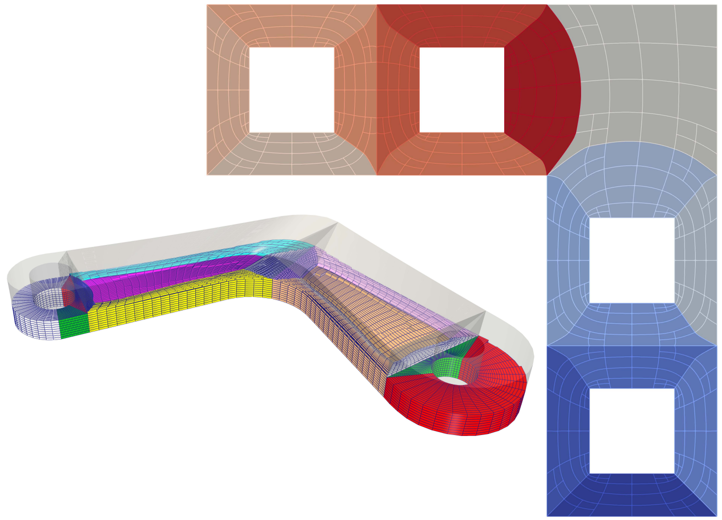
Conclusions and outlook

- **G+Smo offers** three major classes of **parameterization methods**;
- Demonstrates **improved robustness and efficiency** over existing methods;
- Applicability in both **academic** and **real-world industry scenarios**;

- Future Work:
  - Exploring **AI techniques** for geometric modelling and design-through-analysis (DTA) workflows;
  - Developing a user-friendly **graphical user interface** to support broader adoption.





Many thanks for your attention!

Q&A

If you are interested in my research, please feel free to contact me! ;-)

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