

Analysis-suitable parameterization for isogeometric analysis

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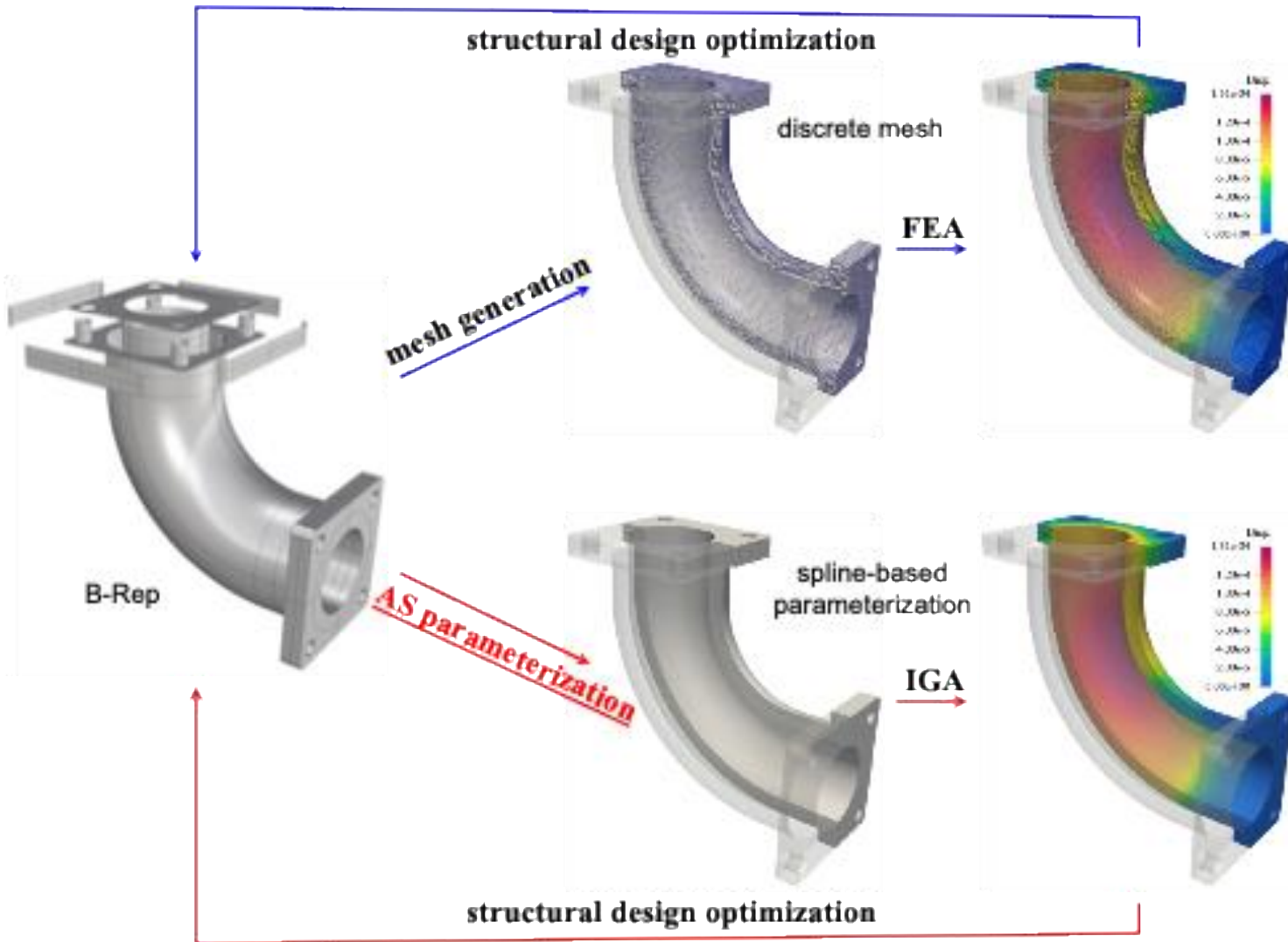
导师: 朱春钢教授(大连理工大学)、Matthias Möller (TU Delft)

Outline

1. Background and motivation
2. Overview of the algorithms
3. Applications
4. Conclusions and outlook

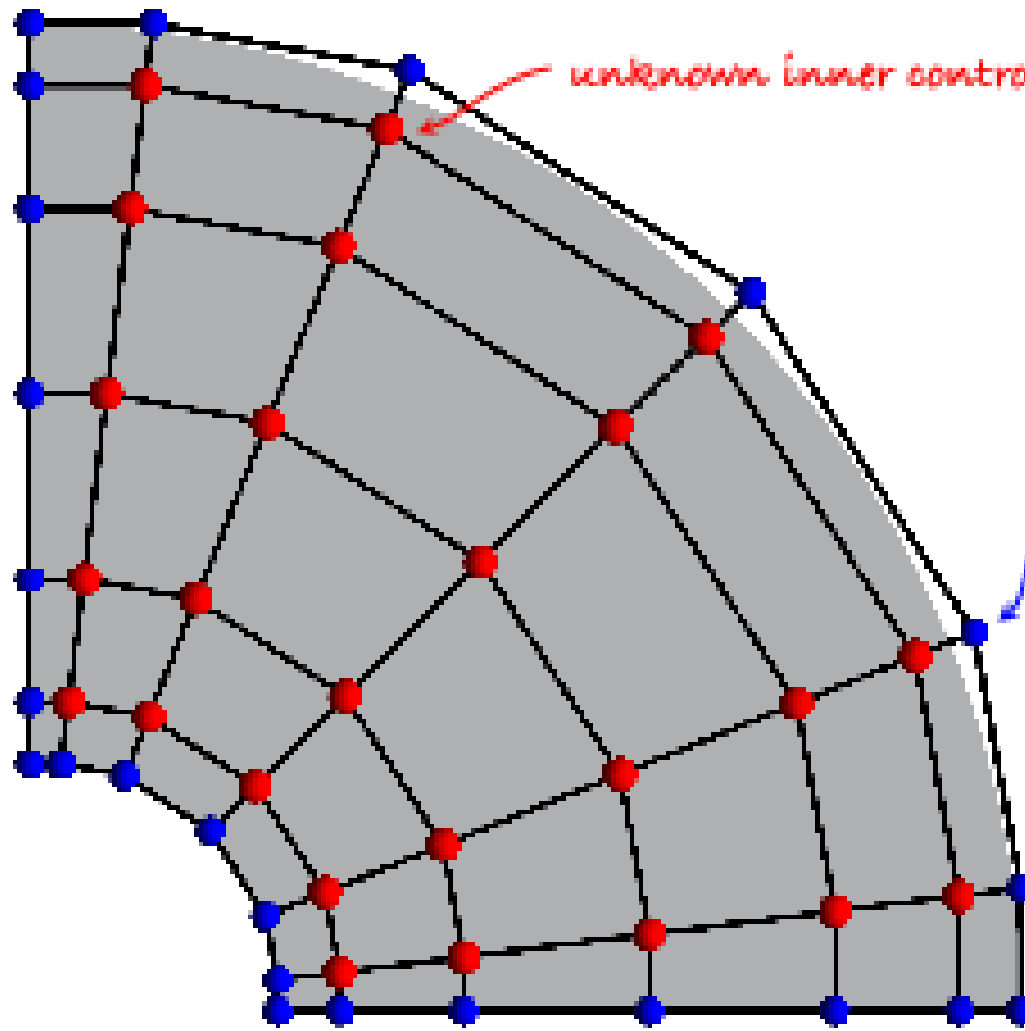
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Design-Through-Analysis product development workflow

- Introduced by T.J.R. Hughes et al., 2005.
- **Key Idea:** Approximate physical fields with **the same basis functions** used in CAD model representation.
- Benefits:
 - **Unified design and analysis;**
 - **Precise, efficient geometry;**
 - **No data type conversion;**
 - **Simplified mesh refinement;**
 - **Continuous high-order fields;**
 - **Superior approximation.**
- Broad applications: shell analysis, fluid-structure interaction, shape and topology optimization, etc.



unknown inner control points P_i

$$x(\xi) = \underbrace{\sum_{i \in I_I} P_i R_i(\xi)}_{\text{unknown}} + \underbrace{\sum_{j \in I_B} P_j R_j(\xi)}_{\text{known}}$$

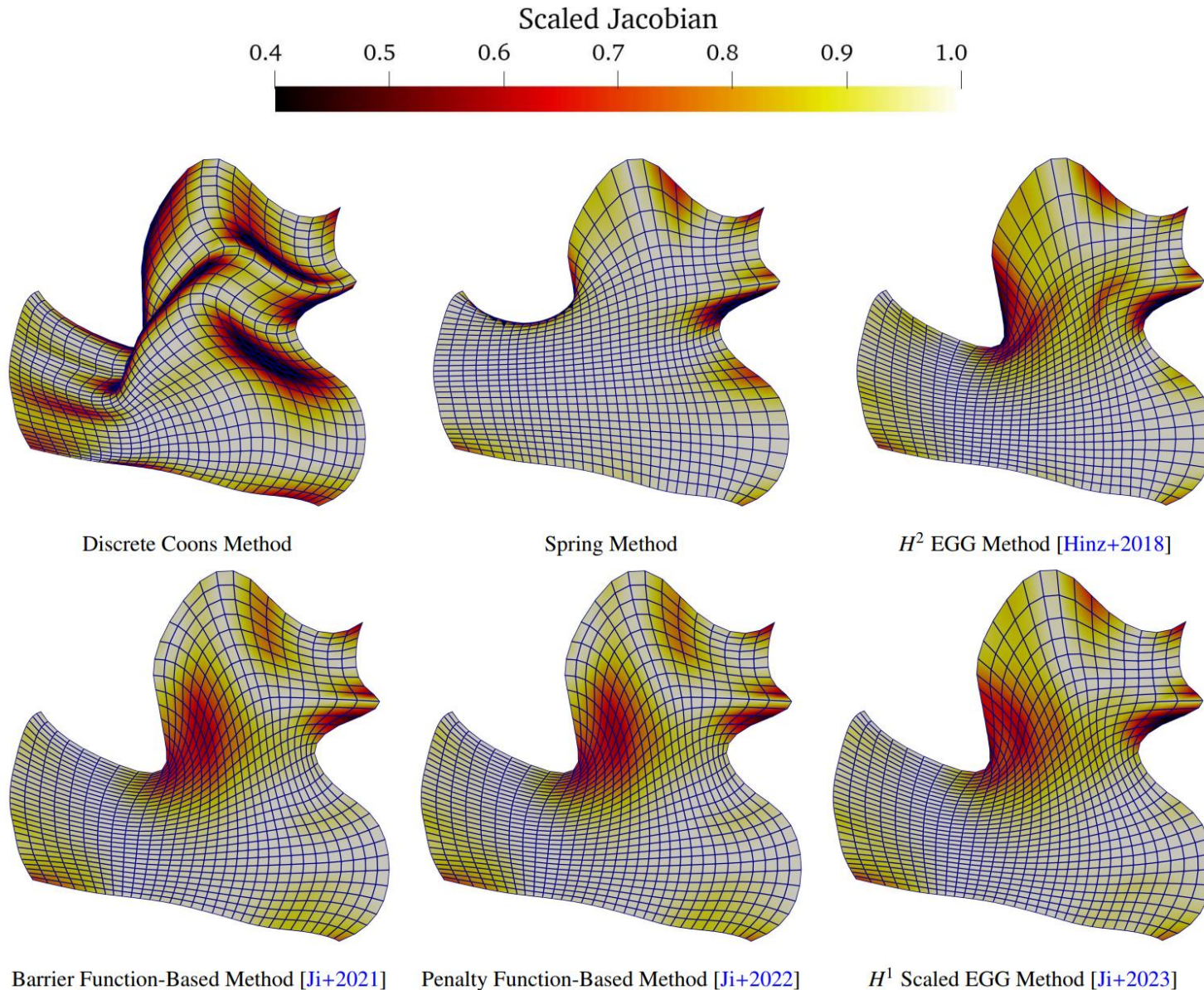
known boundary control points P_j

GOAL: to construct **unknown inner control points P_i** (or basis functions R_i) such that x ensures **bijection** and exhibits **optimal orthogonality** and **uniformity**.

➤ **Parameterization quality** significantly affects **downstream analysis!**

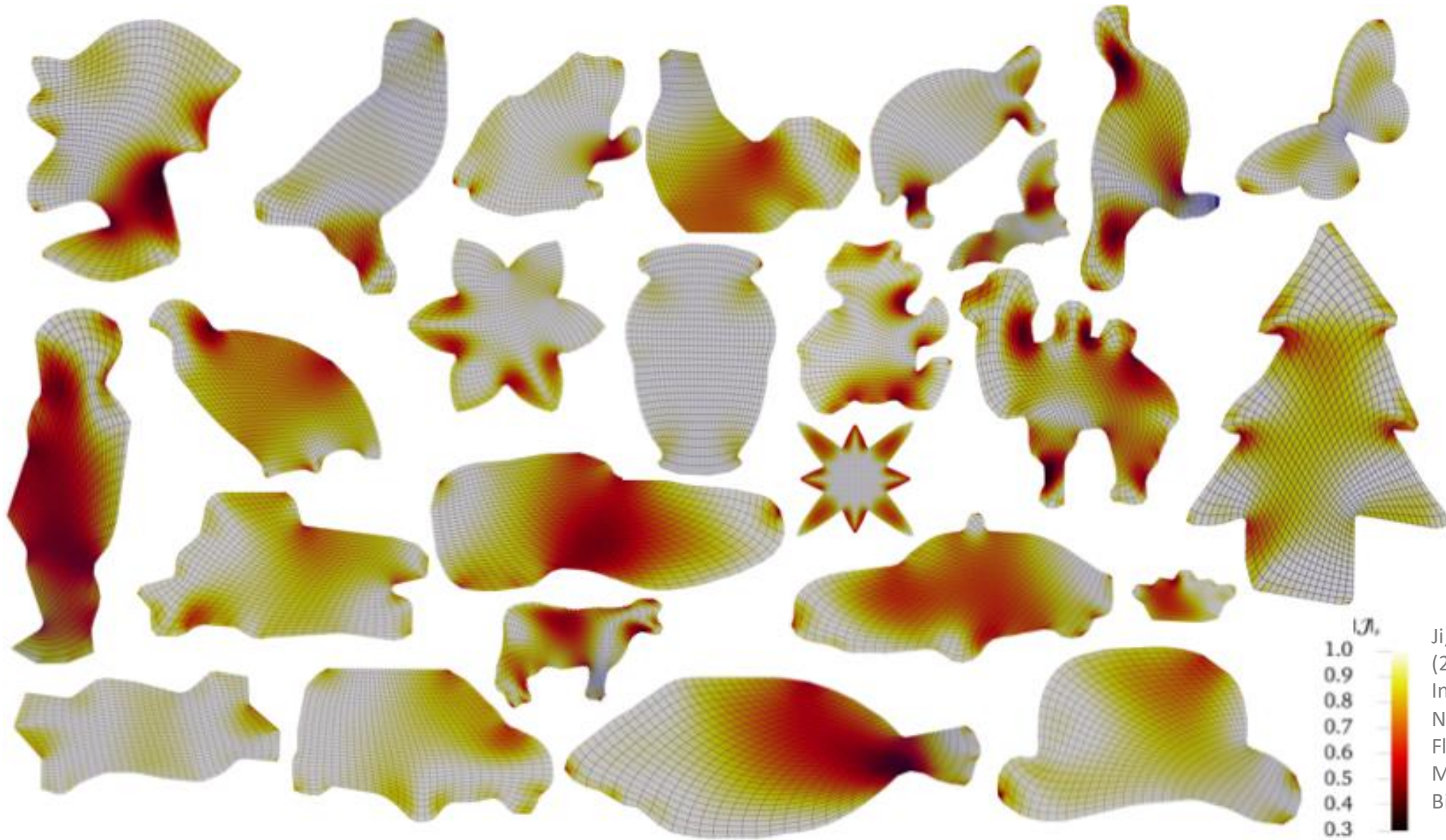
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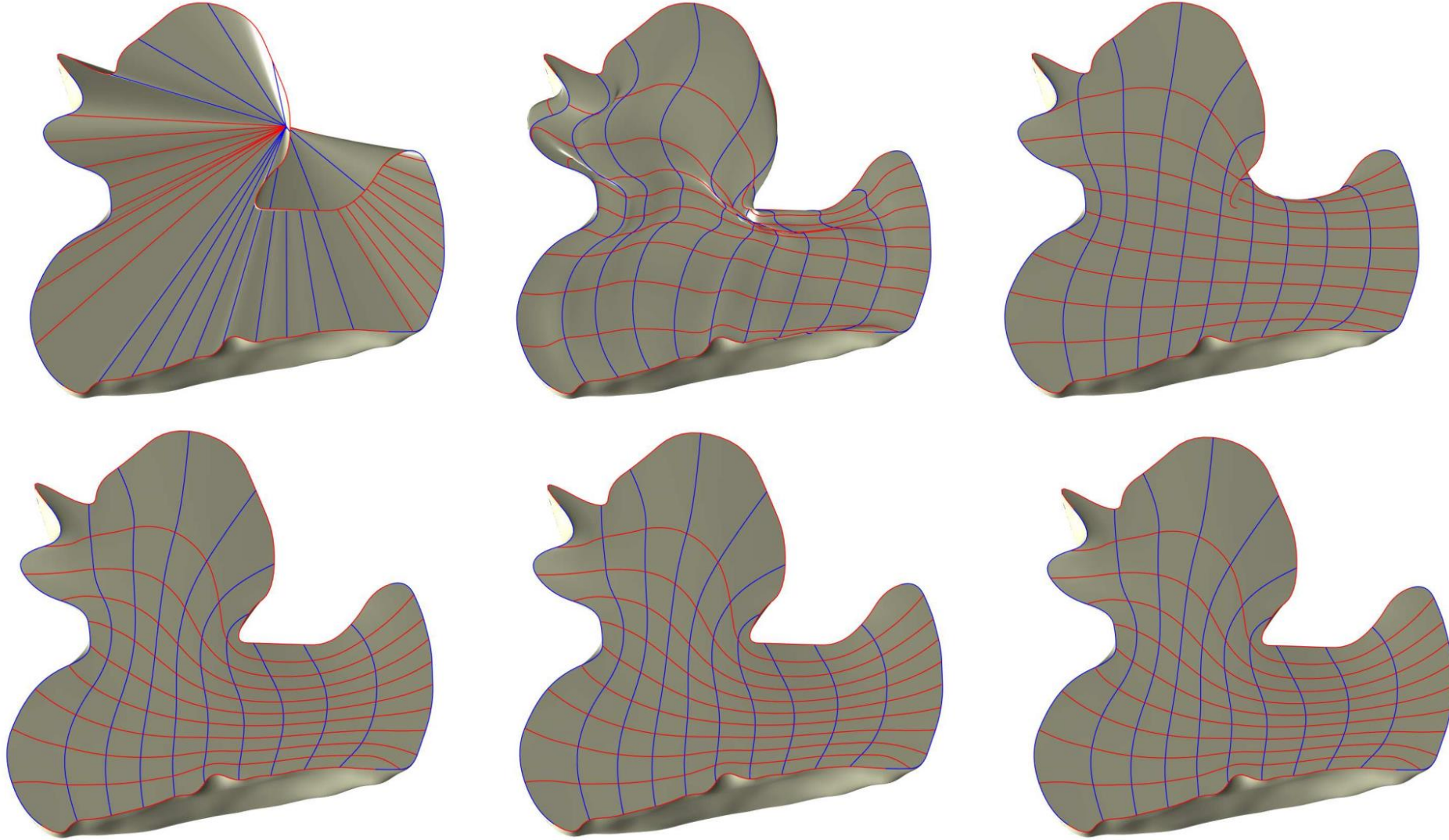


- Algebraic methods:
 - Coons patch
 - Spring patch
- Optimization-based methods:
 - Barrier-function-based ¹
 - Penalty-function-based ²
- PDE-based methods:
 - Elliptic grid generation ³
 - Improved EGG ⁴

1. Ji, Y. et al. (2021). JCAM, 396, 113615.
2. Ji, Y. et al. (2022). CAGD, 94, 102081.
3. Hinz, J. et al. (2018). CAGD, 65, 48-75.
4. Ji, Y. et al. (2023). CAGD, 102, 102191.



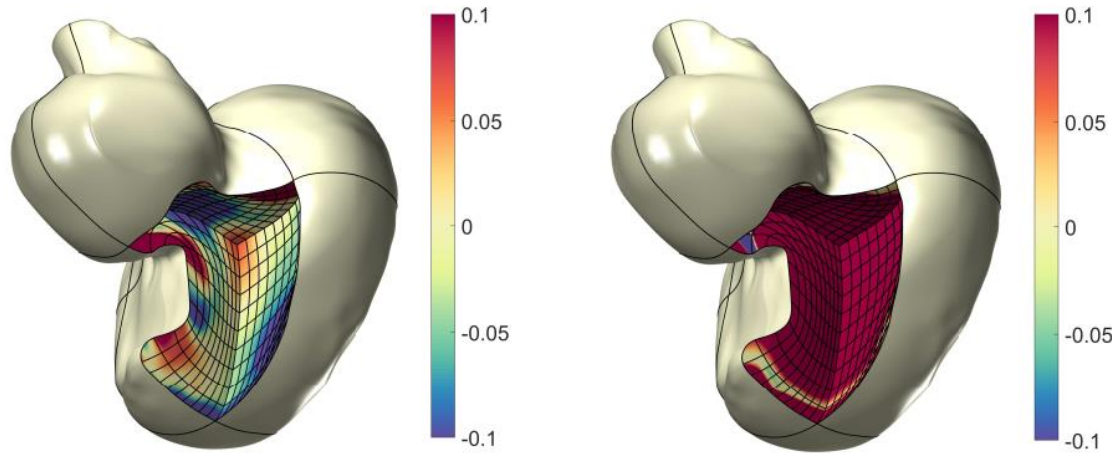
Ji, Y., Möller, M., Verhelst, H.M. (2024). Design Through Analysis. In: Bodnár, T., Galdi, G.P., Nečasová, Š. (eds) Fluids Under Control. Advances in Mathematical Fluid Mechanics. Birkhäuser, Cham.



Same point

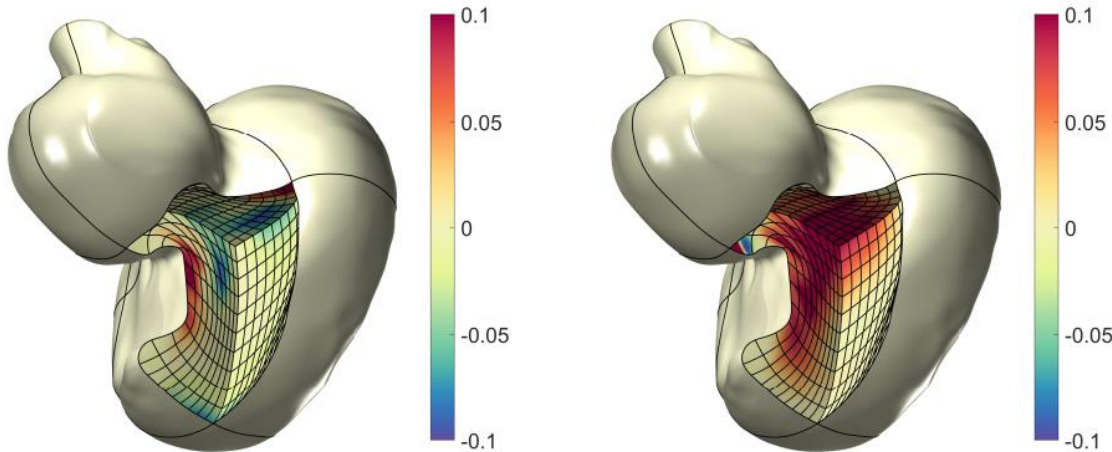
Coons Patch

Smoothness energy



$$m_{SJ}^{Ours} - m_{SJ}^{Pan}$$

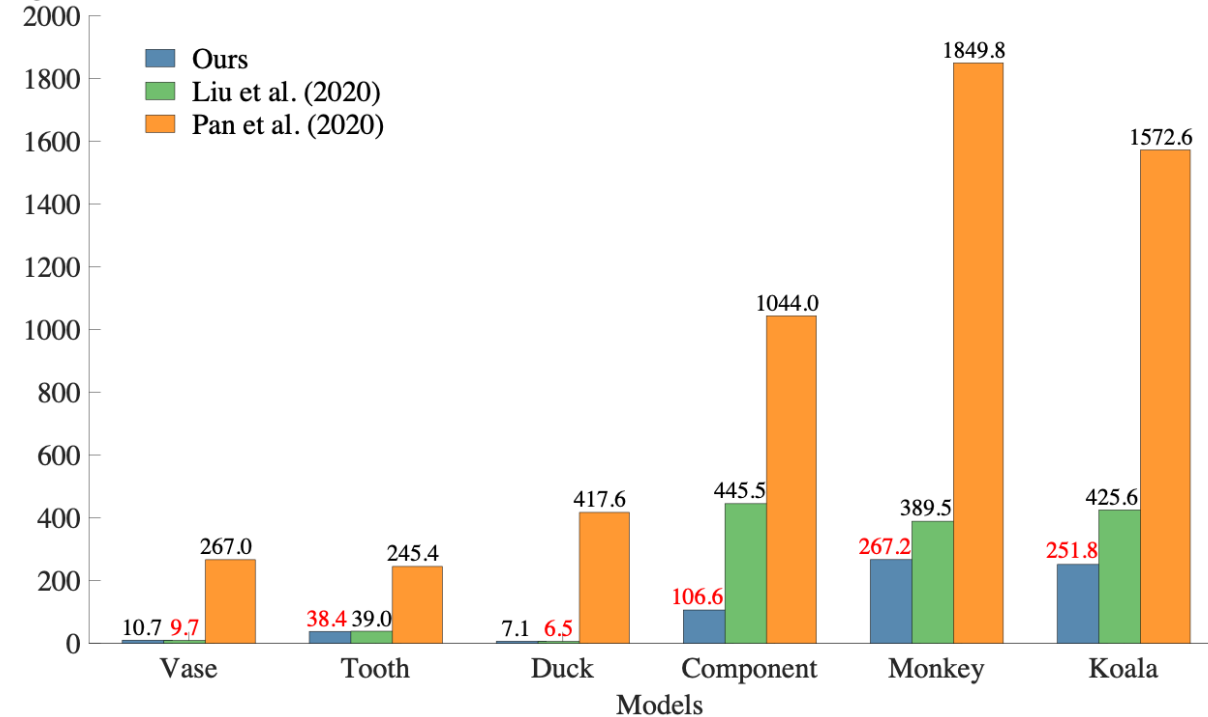
$$m_{unif.}^{Pan} - m_{unif.}^{Ours}$$



$$m_{SJ}^{Ours} - m_{SJ}^{Liu}$$

$$m_{unif.}^{Liu} - m_{unif.}^{Ours}$$

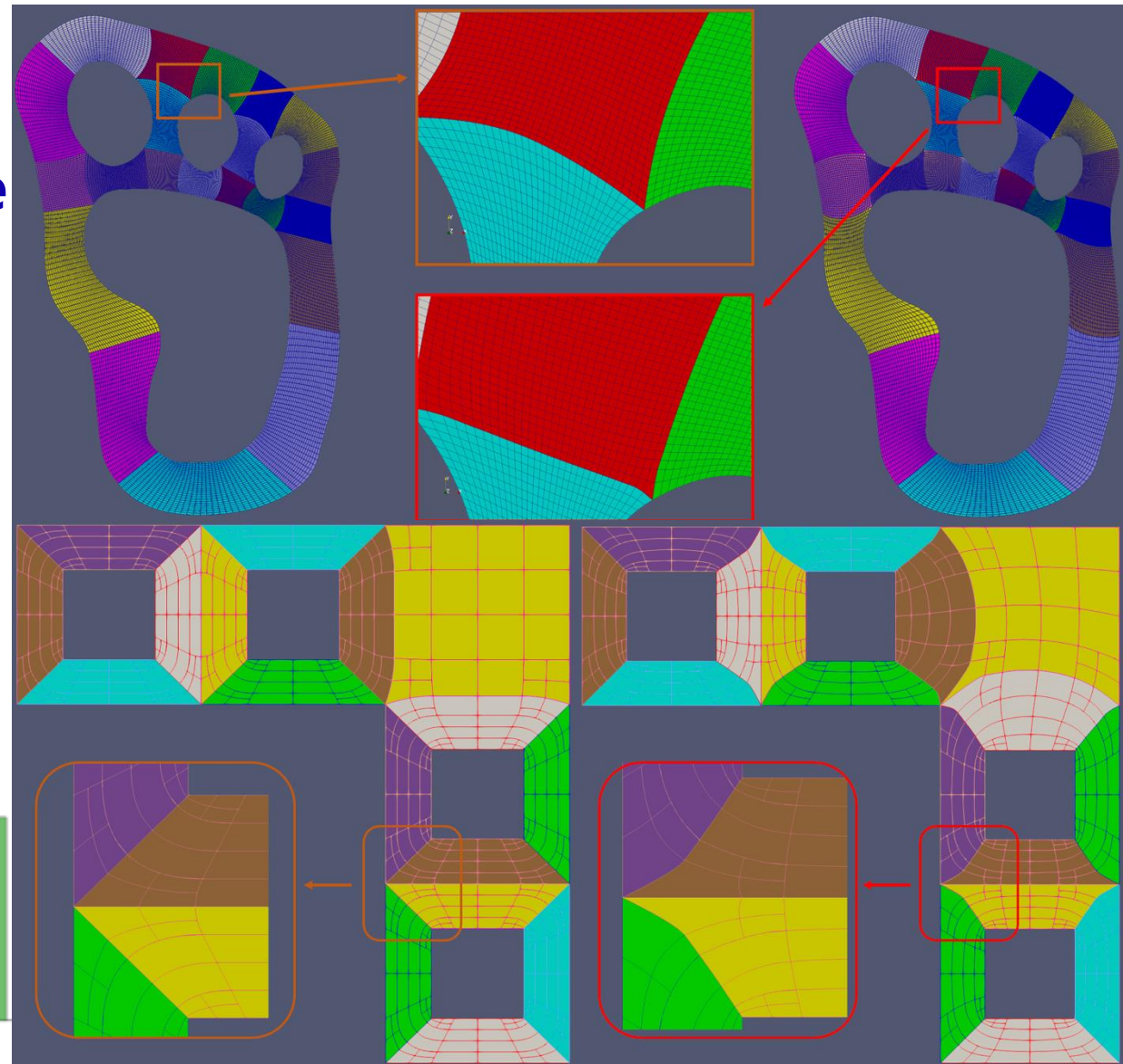
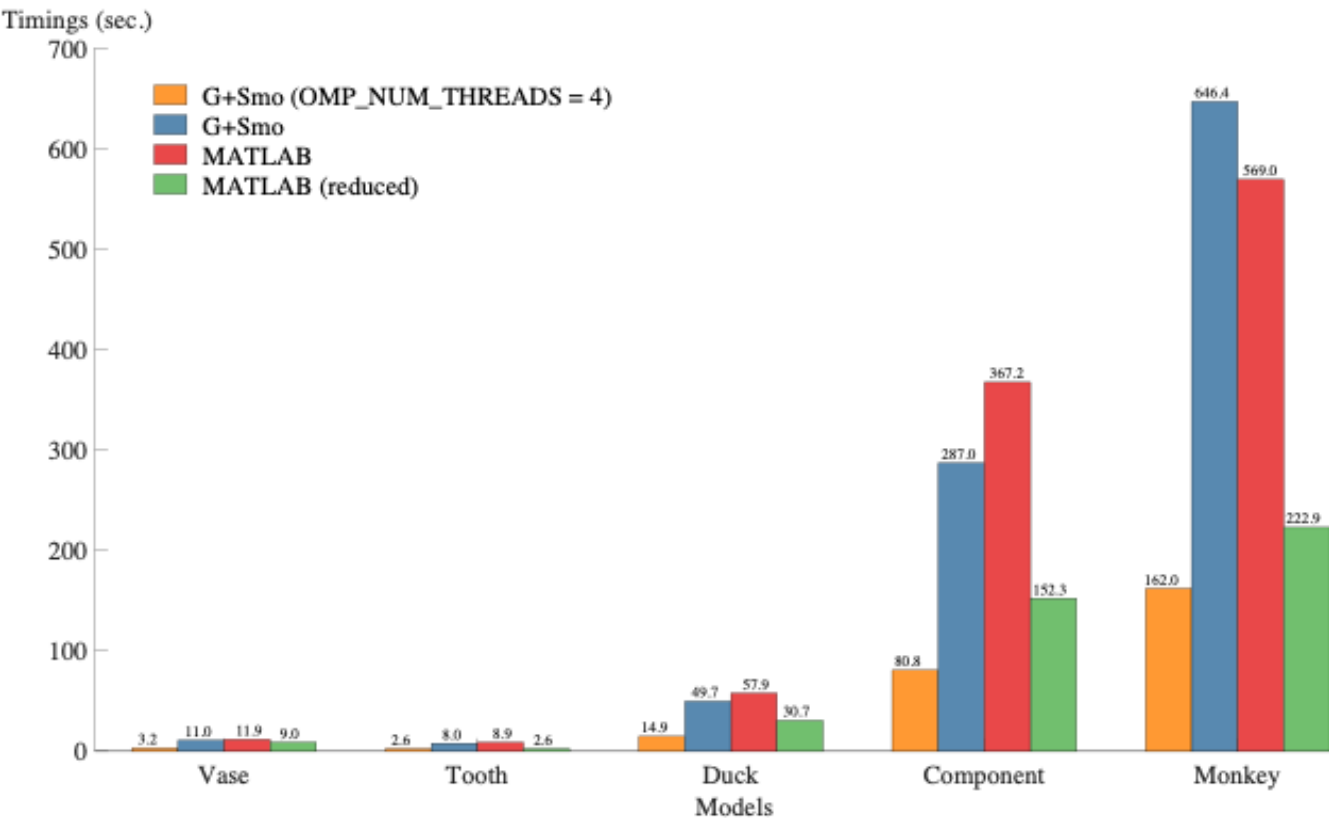
Timings (sec.)

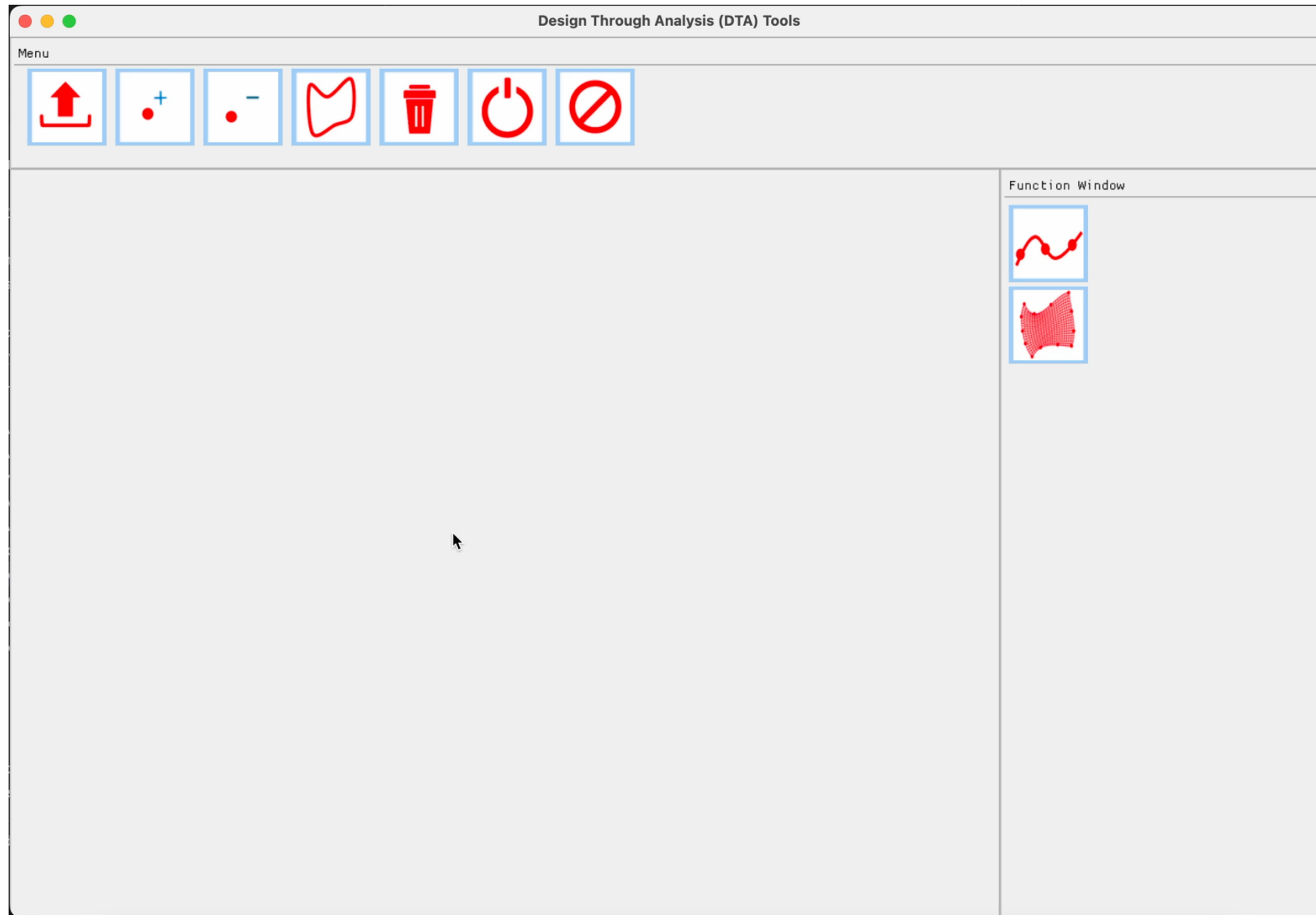


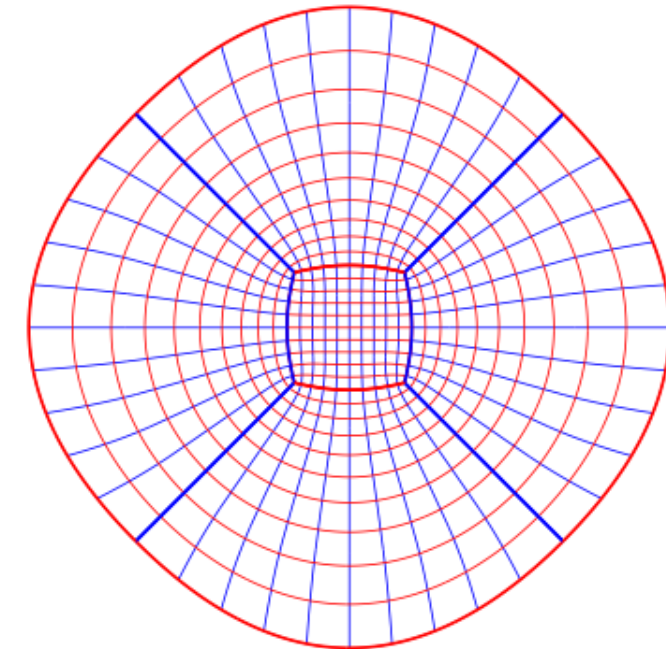
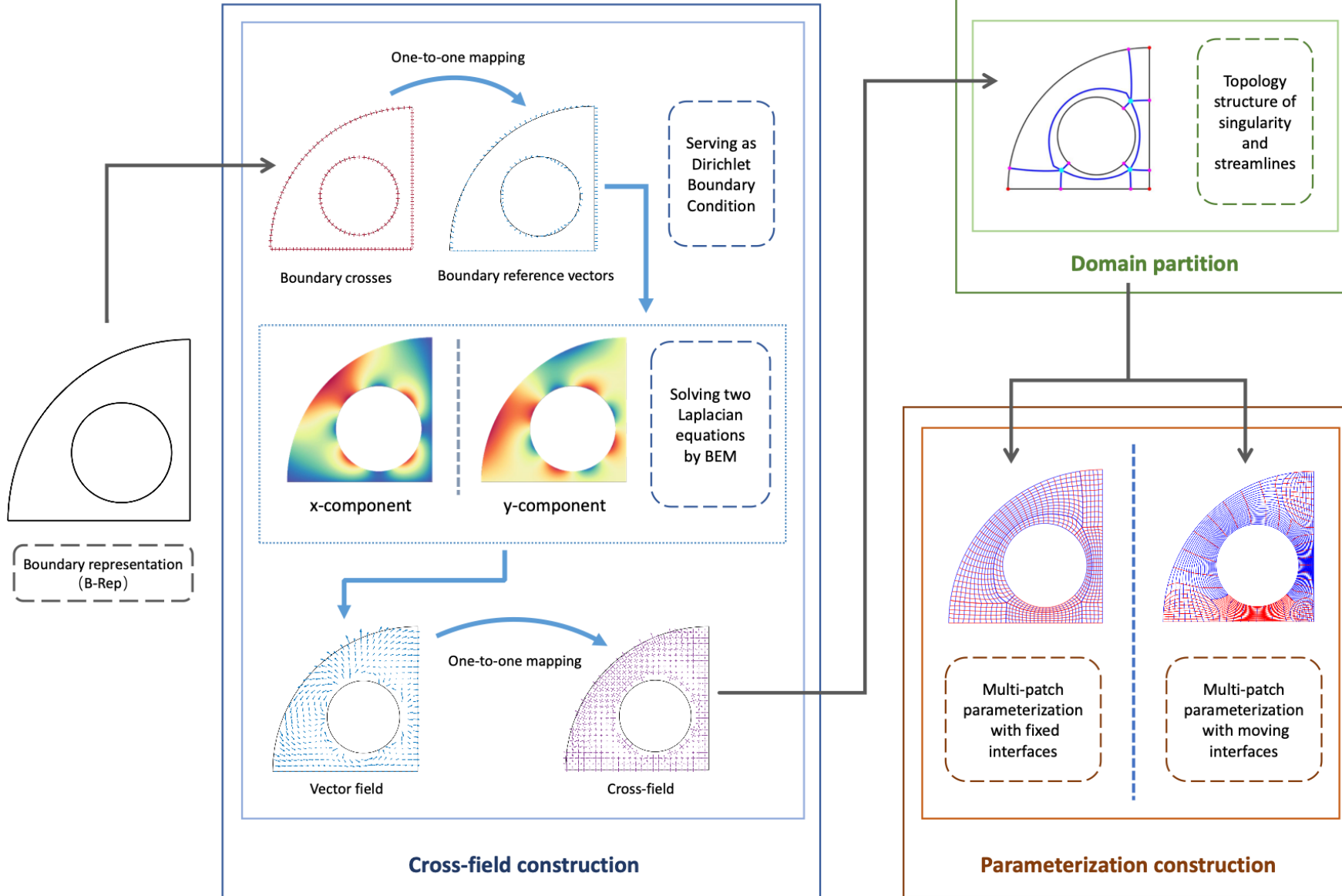
- **Positive values** indicate our method performs better;
- Efficiency comparison (MATLAB vs. C++ (Pan+ & Liu+):
 - Significantly **faster** than Pan et al. (2020);
 - Large-Scale Models: Outperforms Liu et al. (2020).

1. Pan, M., Chen, F., & Tong, W. (2020). CMAME, 359, 112769.
2. Liu, H., Yang, Y., Liu, Y., & Fu, X. M. (2020). CAGD, 79, 101853.

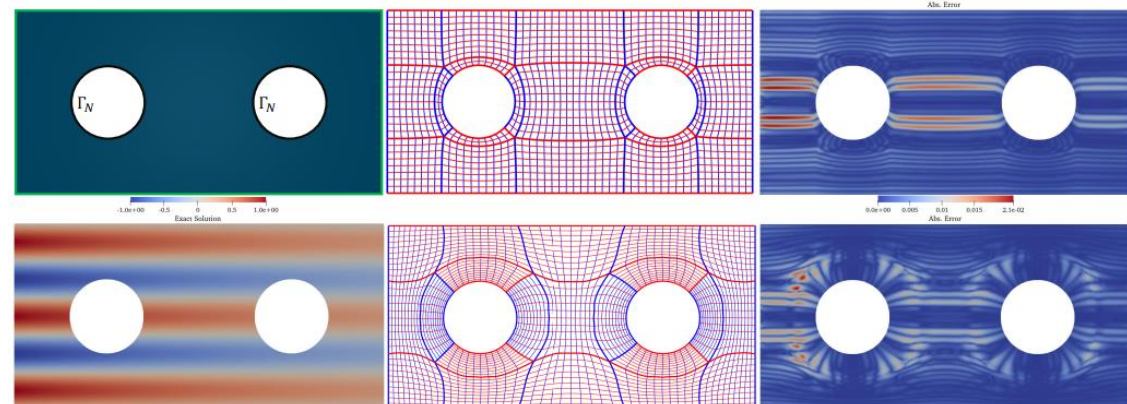
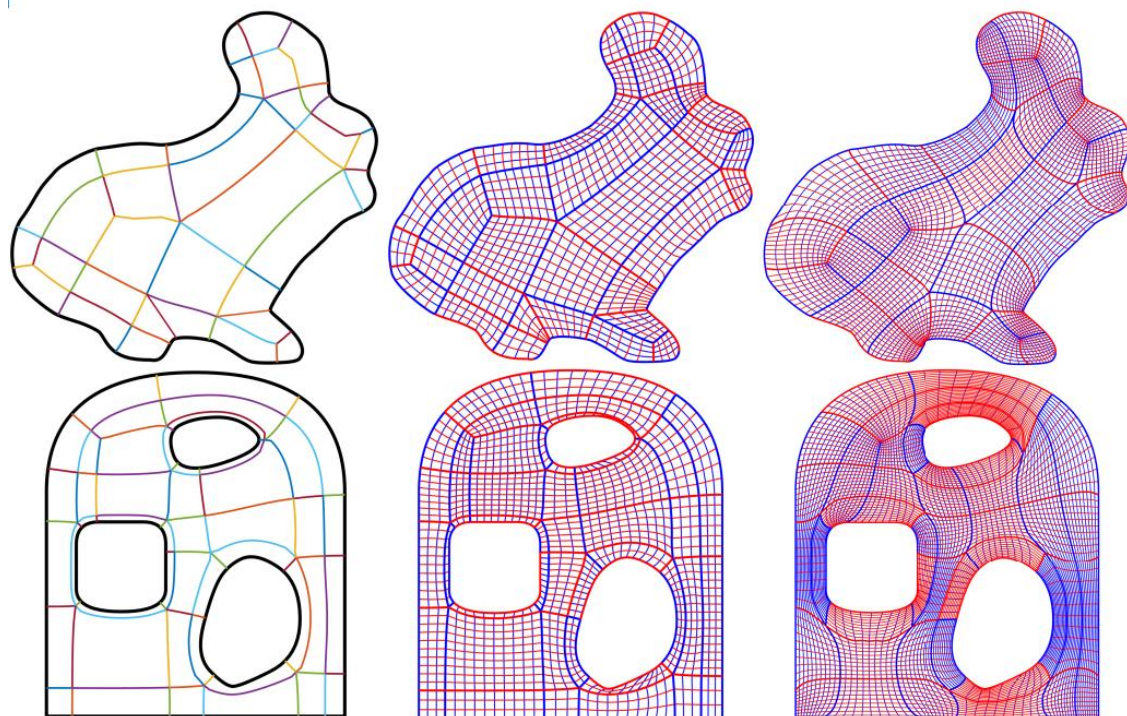
- In our released G+Smo implementation, **3-4x speed-up**;
- Suitable for **multi-patch** and **THB-spline** parameterization;



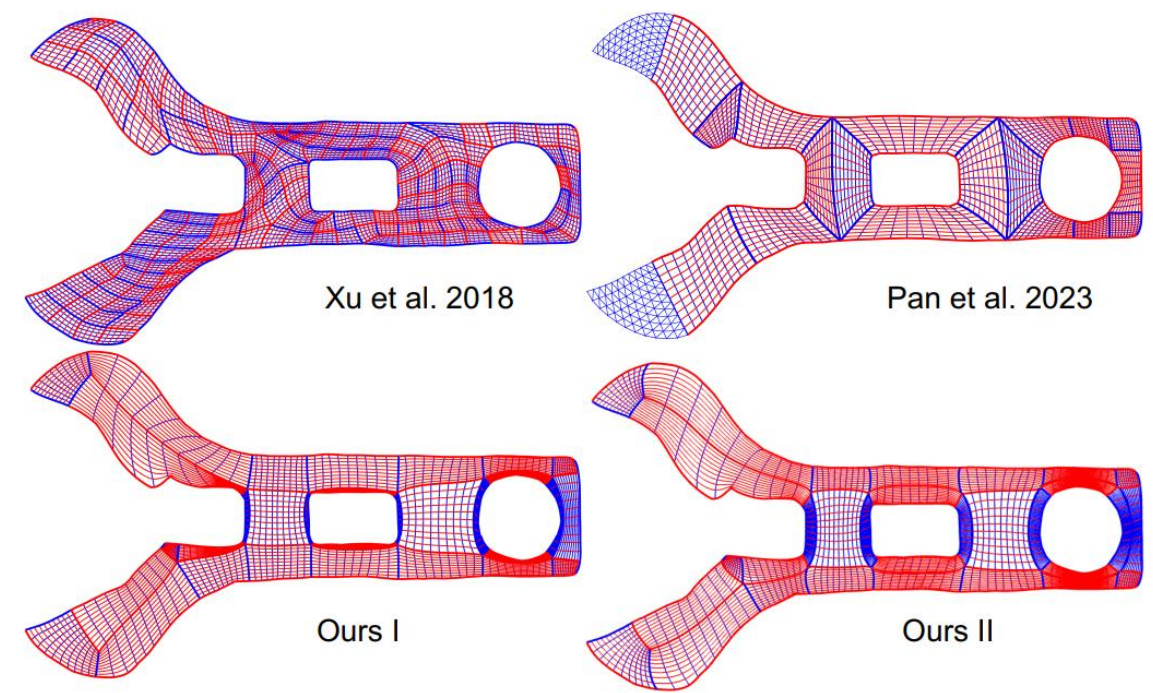


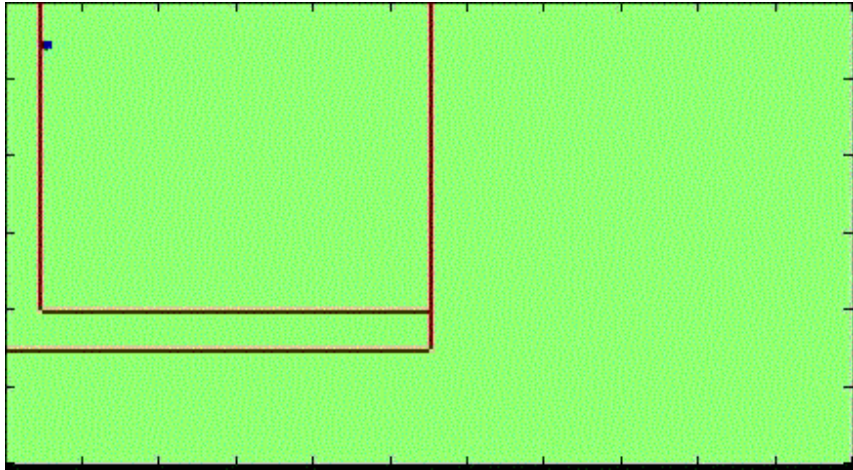


Zhang, Y., Ji, Y., & Zhu, C. G. (2024). Multi-patch parameterization method for isogeometric analysis using singular structure of cross-field. *Computers & Mathematics with Applications*, 162, 61-78.

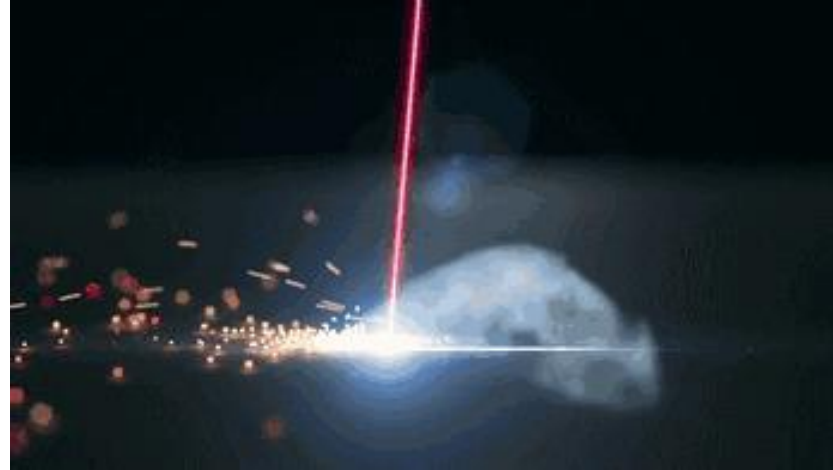


Model	#Patch	Method	$ \mathcal{J} _s$		<i>unif.</i>		
			min.	avg.	min.	avg.	max.
rabbit	33	Coons	-0.8593	0.9628	0.7030	0.9410	1.0524
		fixed-I	0.2204	0.9504	0.6103	0.9544	0.9982
		moving-I	0.02918	0.9283	0.0000	0.9550	1.0000
3 holes	46	Coons	-0.5492	0.9710	0.8008	0.9573	1.0958
		fixed-I	0.1545	0.9716	0.8007	0.9573	0.9978
		moving-I	0.1461	0.9361	0.6791	0.95710	0.9968

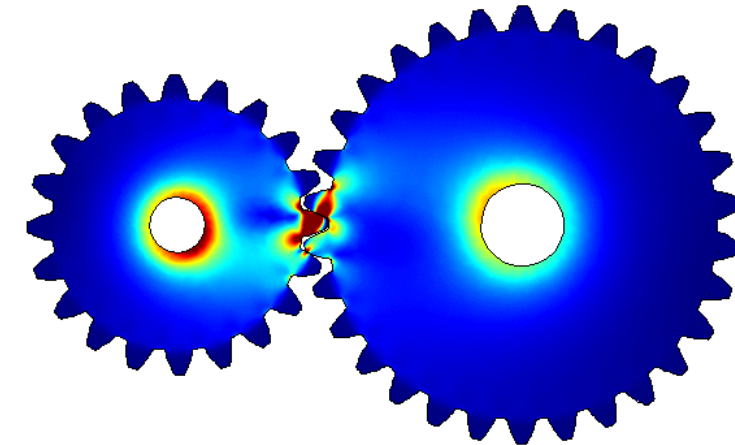




Wave propagation



Laser printing

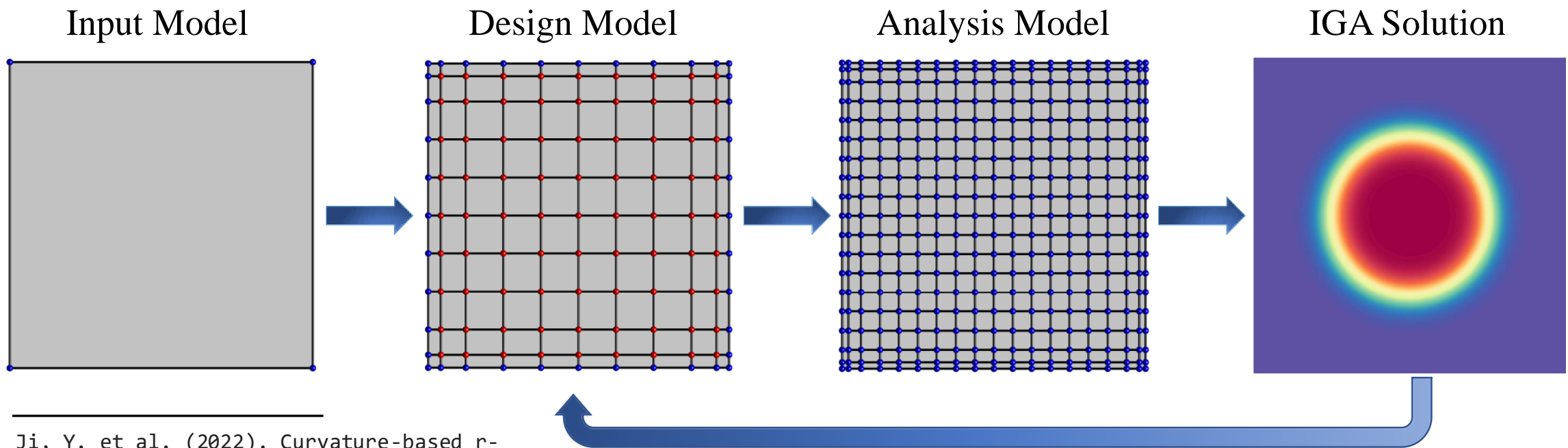


Stress distribution

- **Localized and anisotropic features** extensively exist in physical phenomena;
- **Isotropic parameterizations are not efficient** for such problems;
- Anisotropic parameterizations (r-adaptivity):
 - Enhance per-DOF accuracy while keeping constant total DOFs.
 - Keep the sparse pattern of mass matrix and stiffness matrix.

Anisotropic parameterizations are often solution-dependent:

- Need **good numerical solution accuracy** to drive parameterization;
- Adjust as few control points as possible **for high efficiency**;
- **Bi-level strategy**: a coarse level (design model) to update the parameterization for efficiency and a fine level (analysis model) to perform analysis for accuracy.

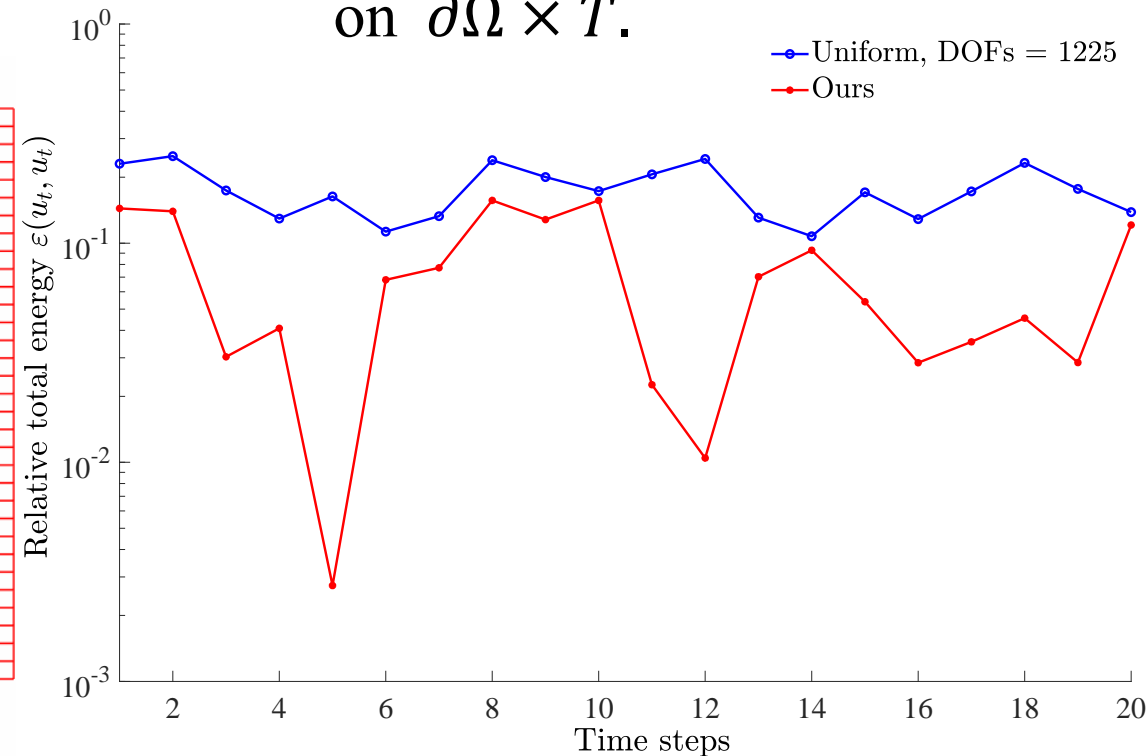
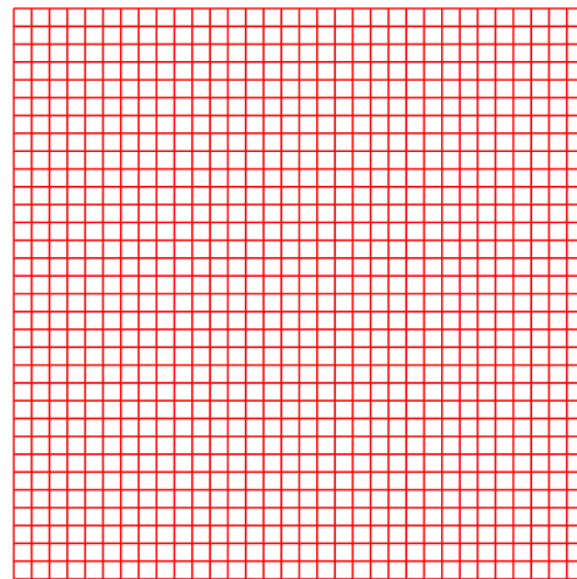
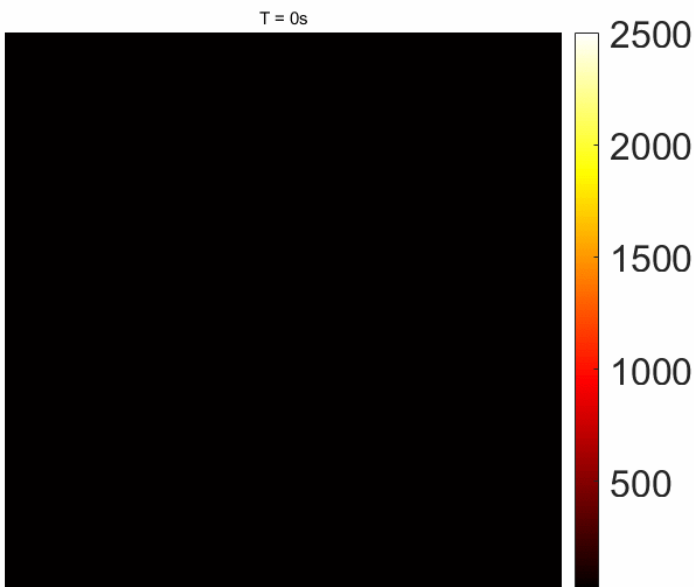


Ji, Y. et al. (2022). Curvature-based r-adaptive planar NURBS parameterization method for isogeometric analysis using bi-level approach. *Computer-Aided Design*, 150, 103305.

Objective function and sensitivity analysis

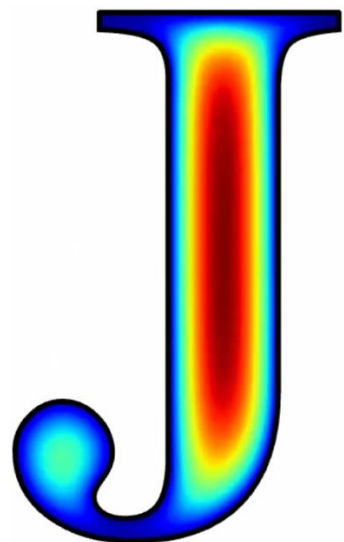
- Consider a two-dimensional linear heat transfer problem with a moving Gaussian heat source:

$$\begin{cases} C_p \rho \frac{\partial u(\mathbf{x}, t)}{\partial t} - \nabla \cdot (\kappa \nabla u(\mathbf{x}, t)) = f(\mathbf{x}, t), & \text{in } \Omega \times T, \\ u(\mathbf{x}, t) = u_0, & \text{in } \Omega, \\ \kappa \nabla u(\mathbf{x}, t) = 0, & \text{on } \partial\Omega \times T. \end{cases}$$

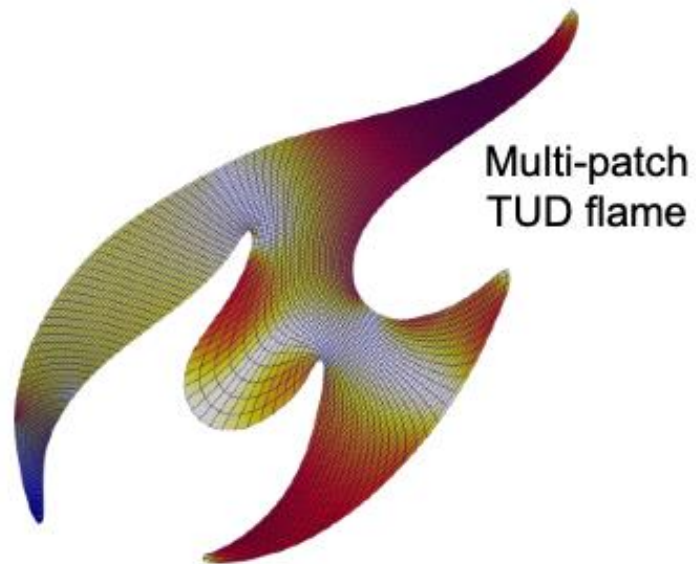


Outline

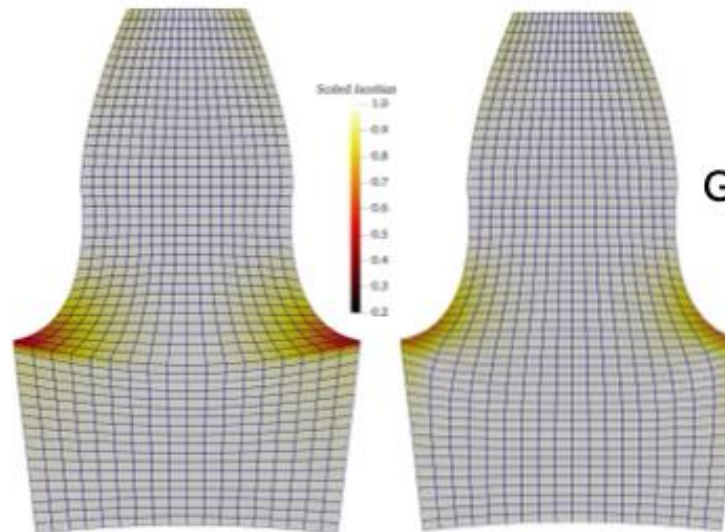
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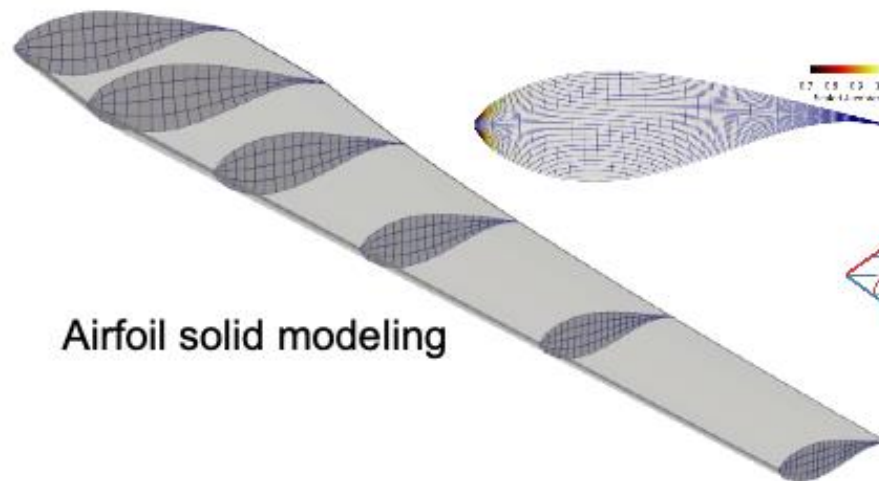
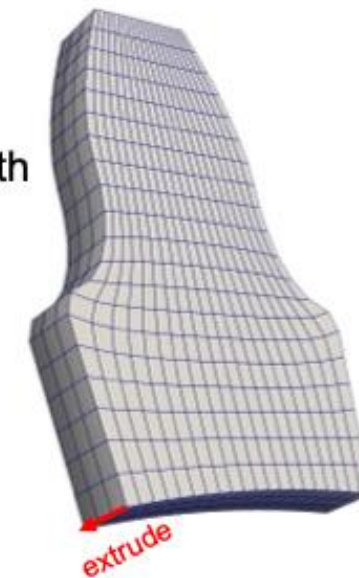
TU Wien



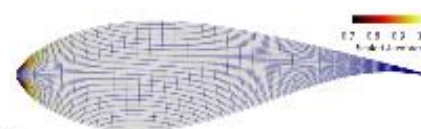
Multi-patch
TUD flame



Gear tooth

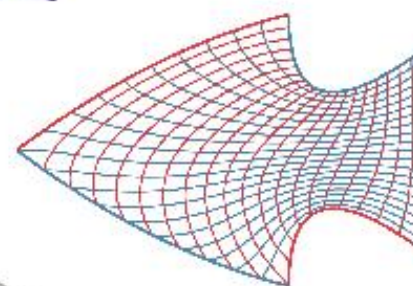


Airfoil solid modeling

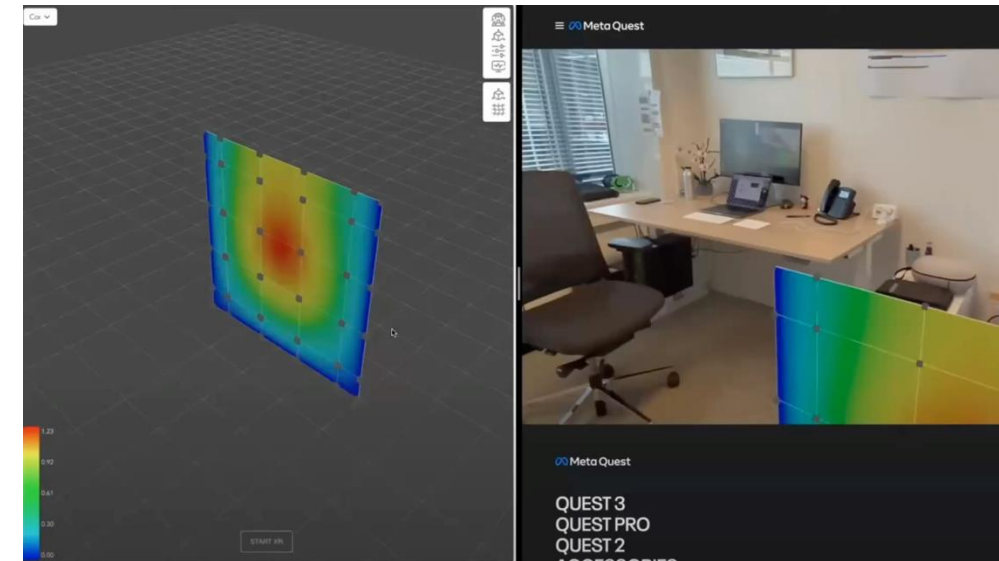
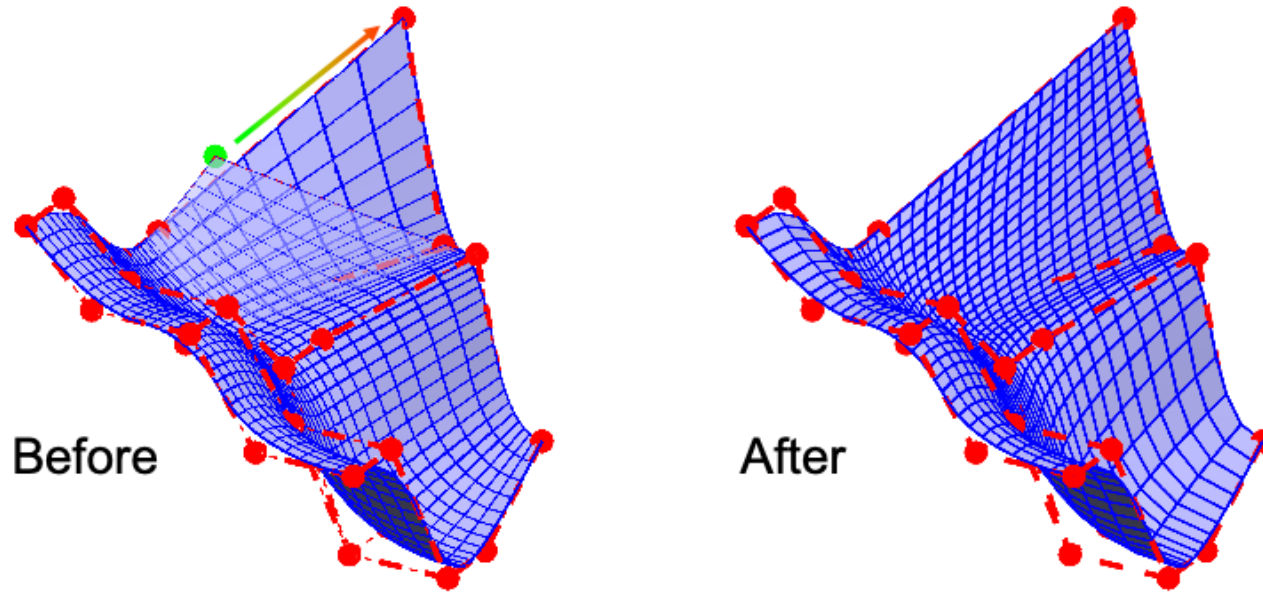
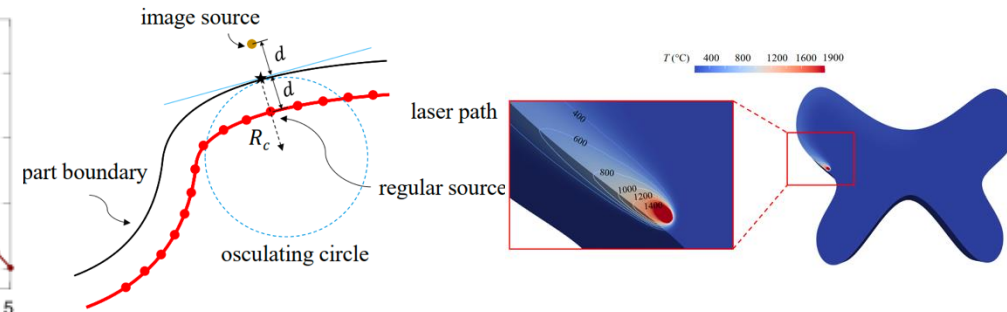
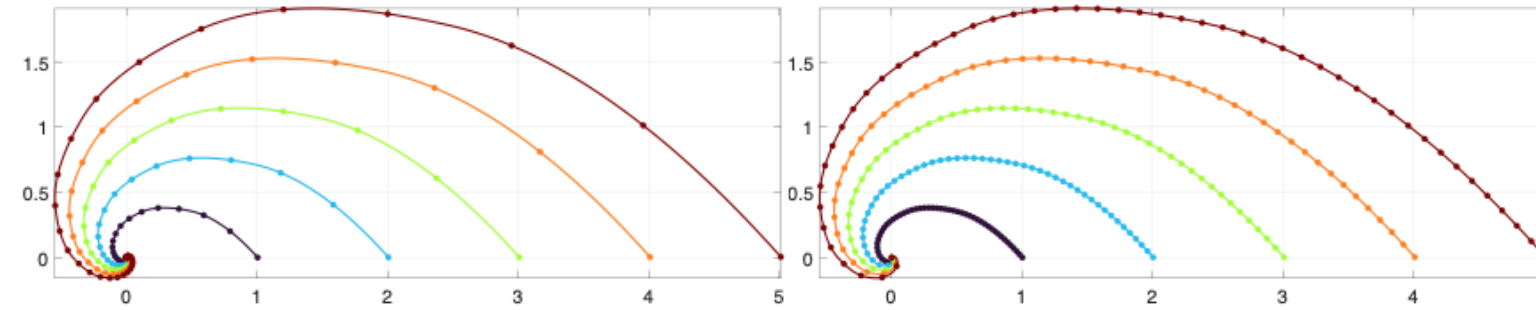


0.2 0.5 0.8 1.0
Scaled Isotopic

Multi-sided surface modeling

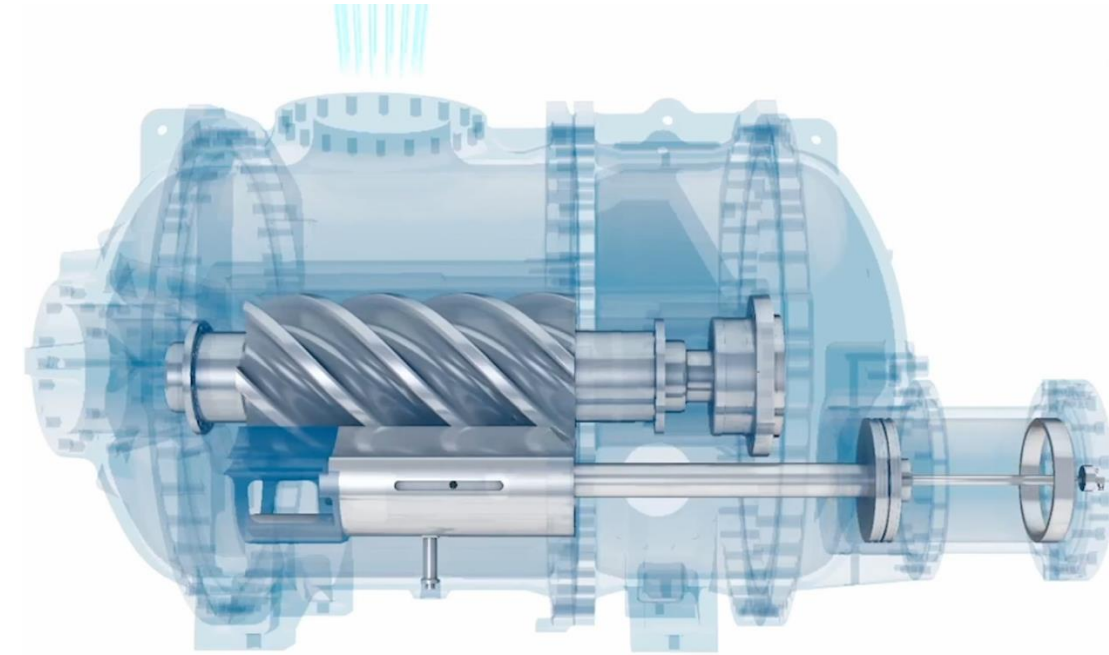


discontinuous stress field (C^0 multi-patch NURBS) **continuous** stress field (multi-sided surface)



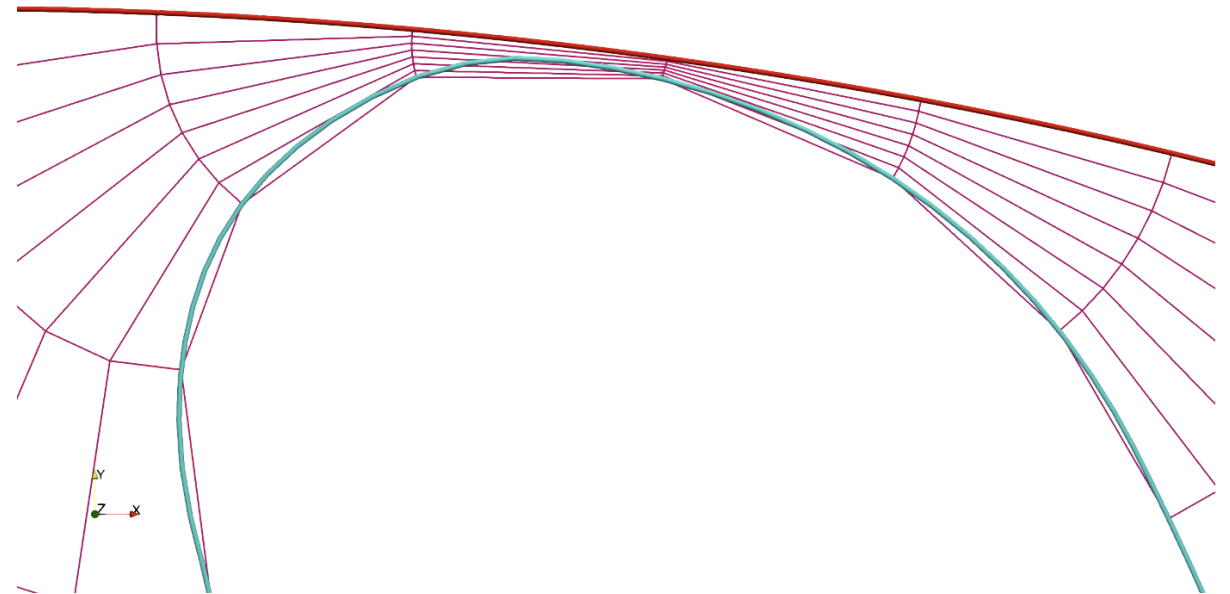
- Curve/surface reparameterization while keeping the geometry;
 - Curve – metal additive manufacturing
 - Surface – VR/AR

Yang, Y., Ji, Y. Möller M, Ayas C. Computational Efficient Process Simulation of Geometrically Complex Parts in Metal Additive Manufacturing, Submitted.



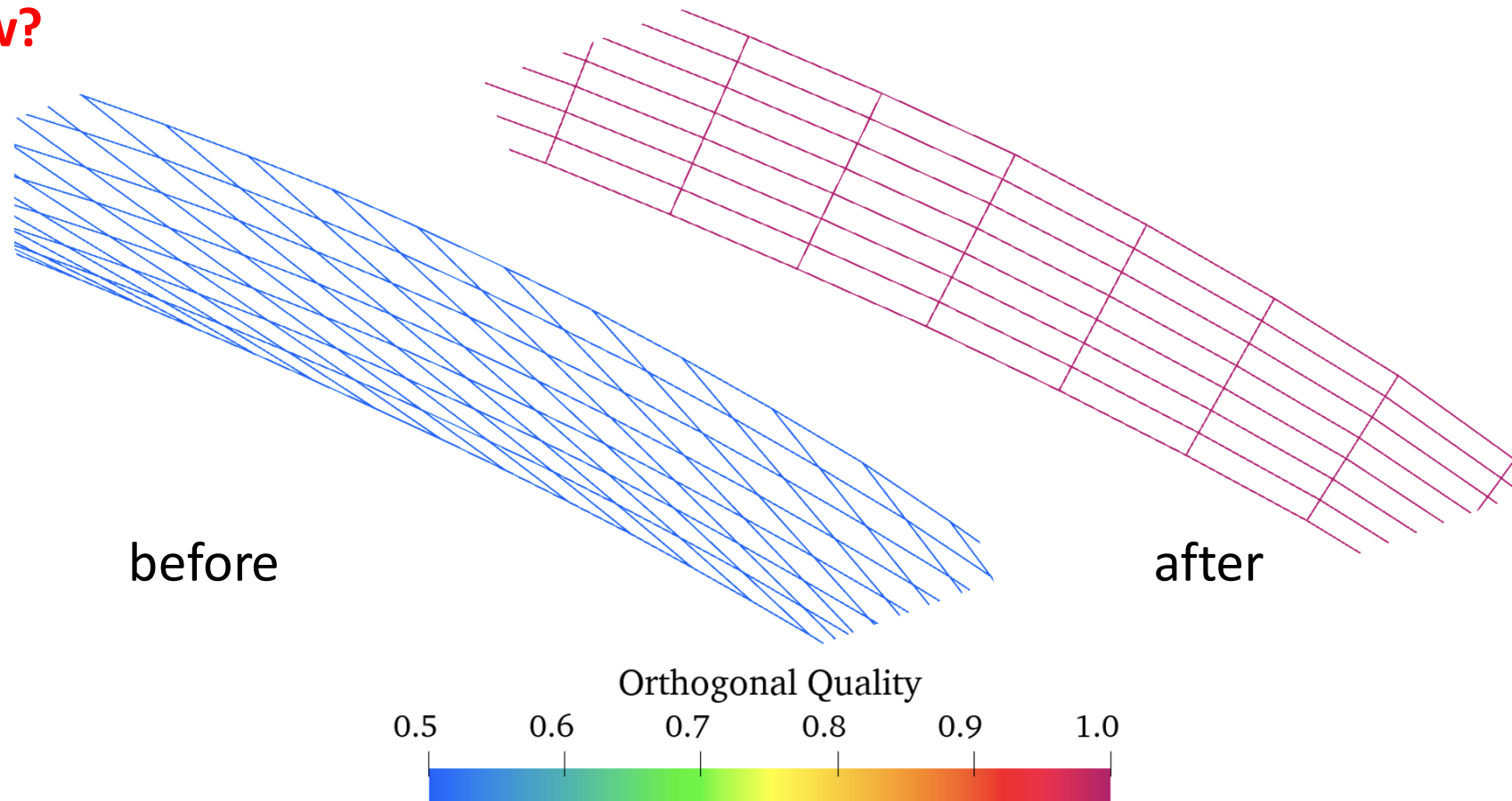
Rotary twin-screw compressor

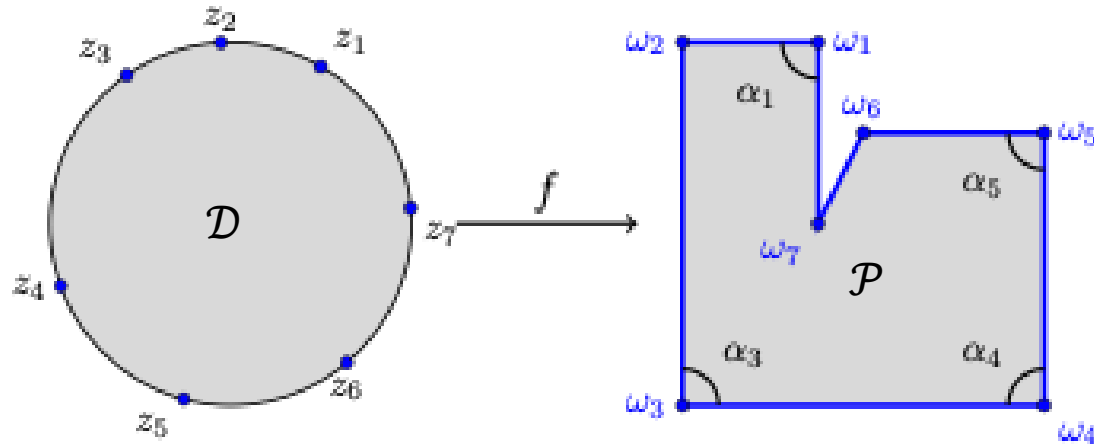
- **Structured mesh generation** is a crucial preprocessing step in the simulation-based analysis of twin-screw machines.
- However, the existing mesh generators typically produce only **linear meshes with straight-sided cells**;
- **Analysis-suitable, high-order NURBS parameterizations.**



Source: <https://www.gascompressors.co.uk/technologies/oil-floodedscrew-compressor/>

- **Parameter speed** of the boundary curves significantly affects the mesh quality;
- Mesh quality is greatly improved by using the **boundary reparameterization technique**.
- **So, how?**

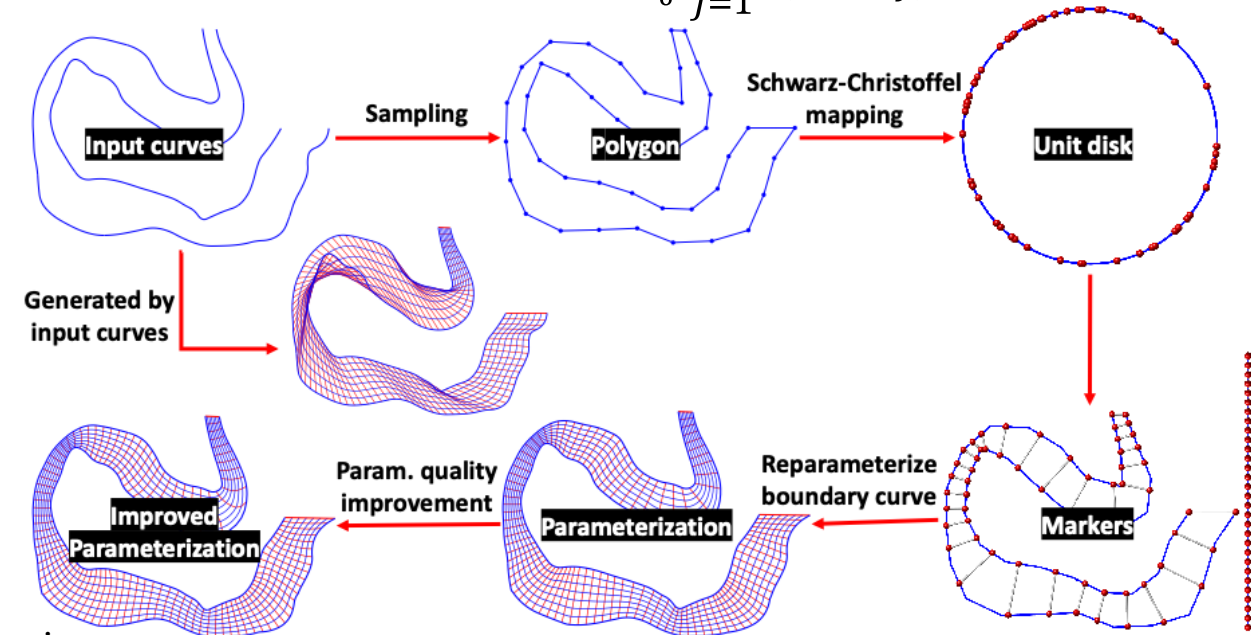




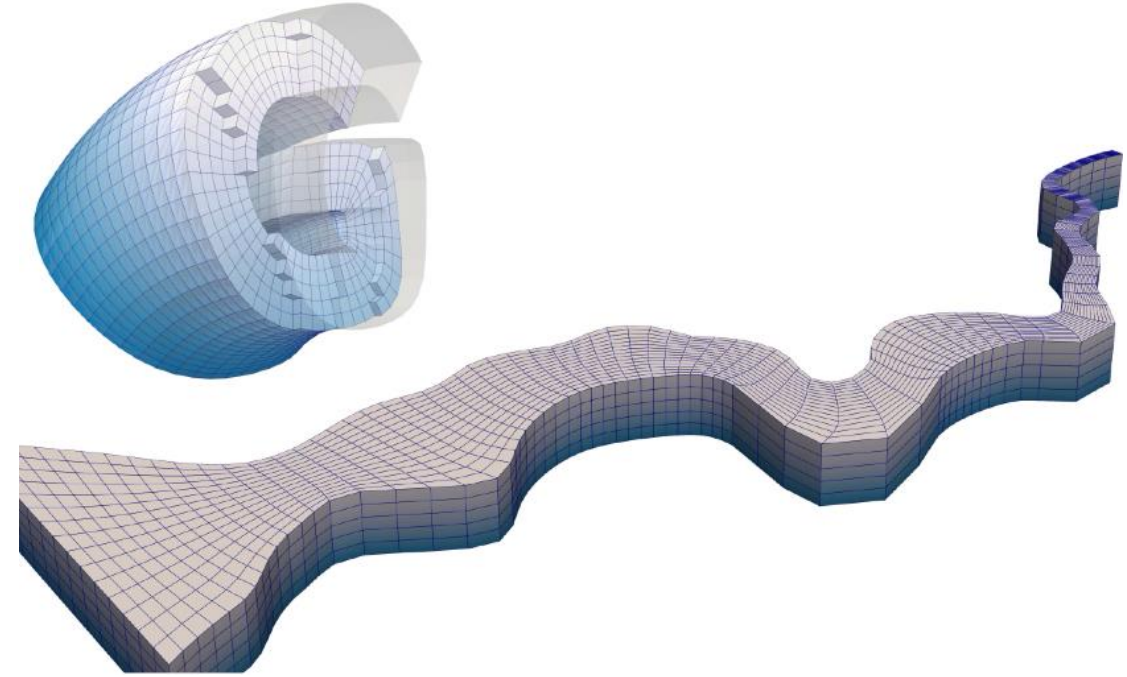
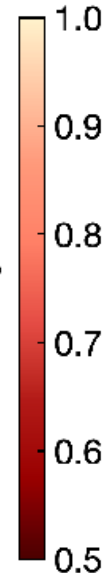
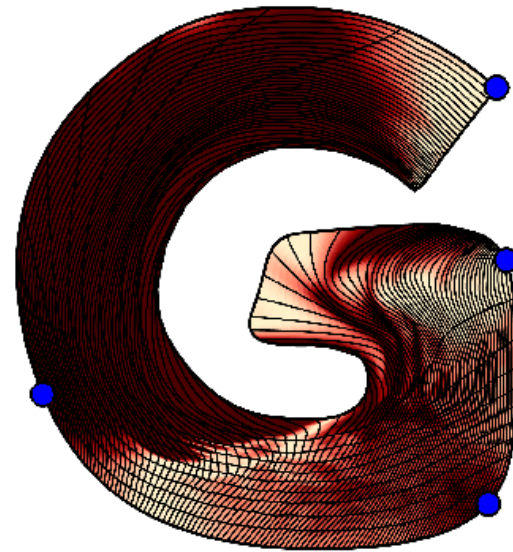
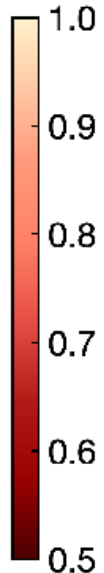
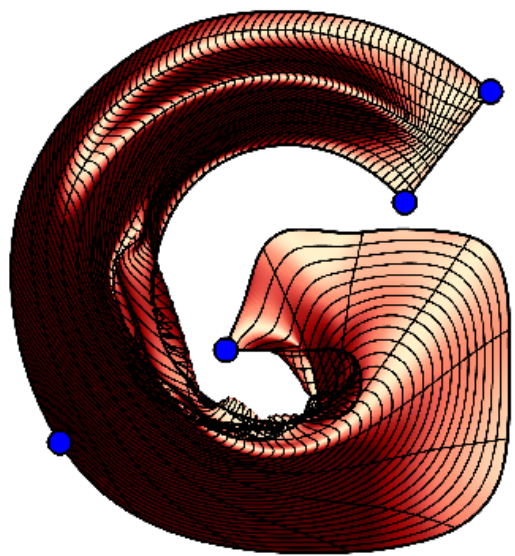
- **Riemann mapping theorem:** \exists analytic function f **with non-zero derivative** such that $f(\mathcal{D}) = \mathcal{P}$.
- **Schwarz-Christoffel formula**

$$f(z) = f(z_0) + C \int_{z_0}^z \prod_{j=1}^n \left(1 - \frac{\zeta}{z_j}\right)^{\beta_j} d\zeta$$

- Solving the Schwarz-Christoffel parameter problem for $\{z_j\}$ numerically allows us to compute sets of markers on the two opposite curves that can be used to reparametrize one curve w.r.t. the other.
- Solving the parameter problem is far from easy, **the CRDT algorithm**^[1] is adopted and implemented.

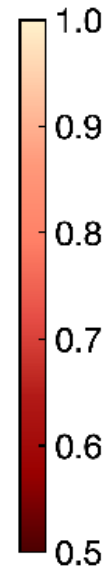
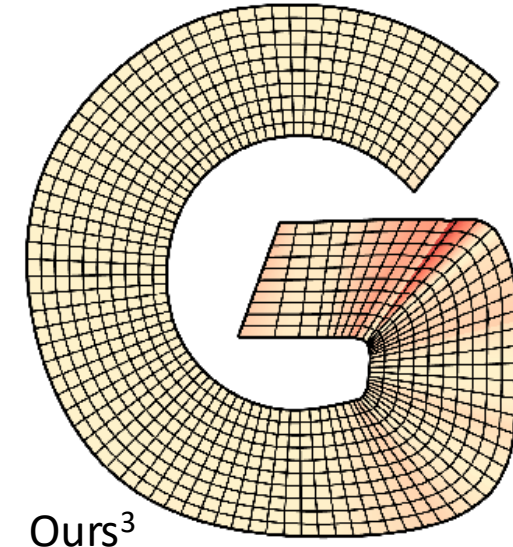
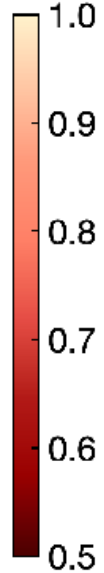
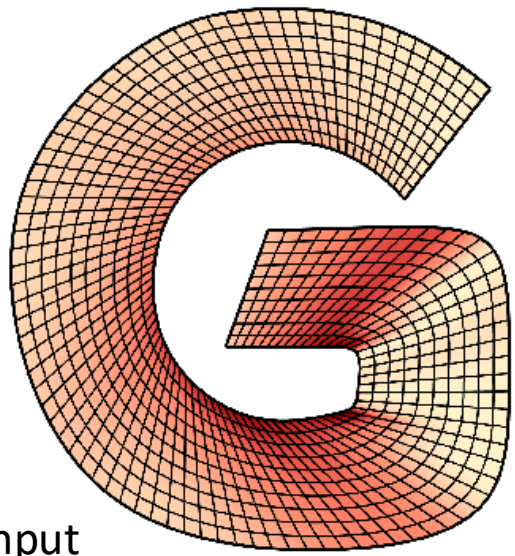


1. Driscoll, T. A., & Vavasis, S. A. (1998). Numerical conformal mapping using cross-ratios and Delaunay triangulation. SIAM Journal on Scientific Computing, 19(6), 1783-1803.



Optimal Transport¹

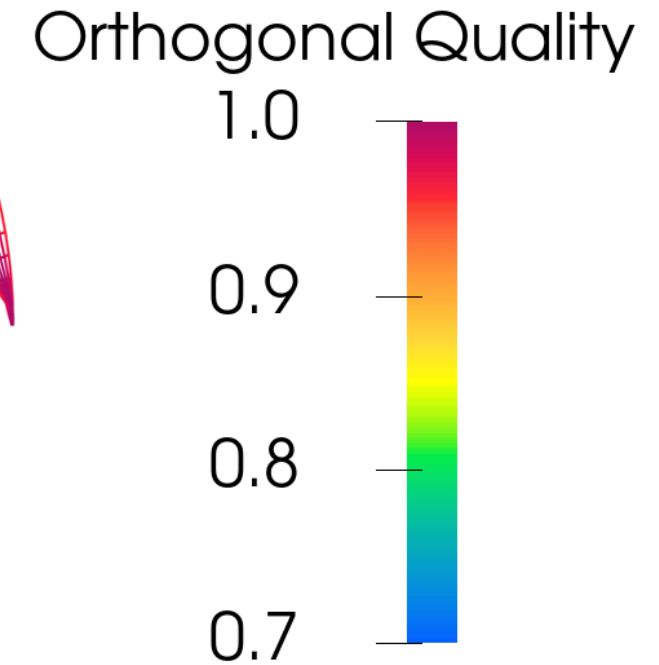
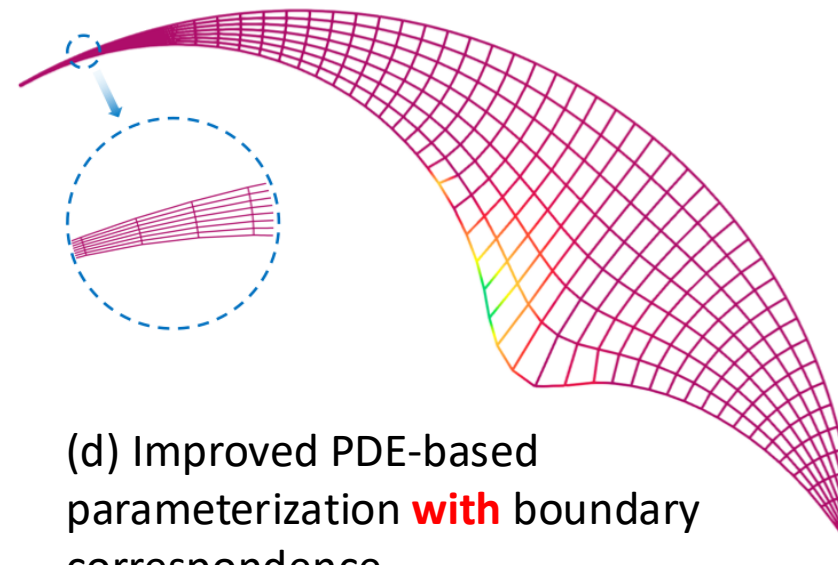
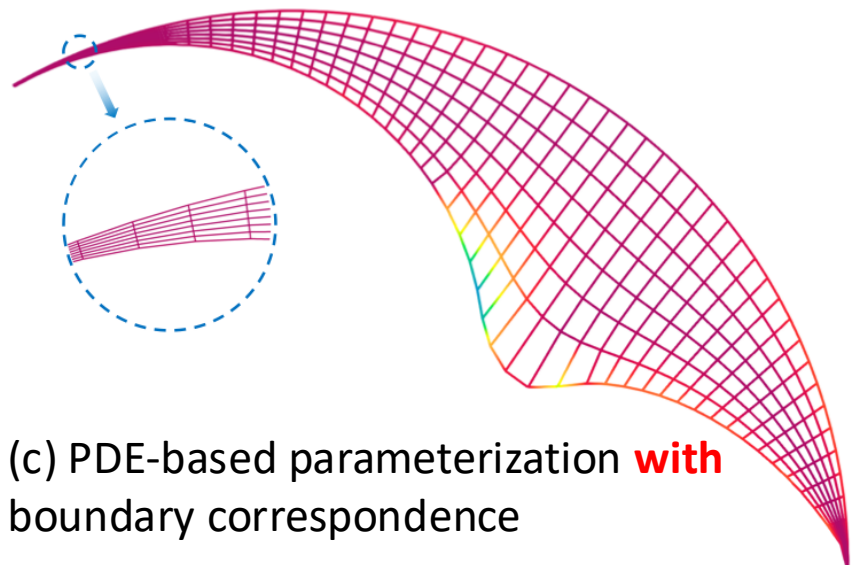
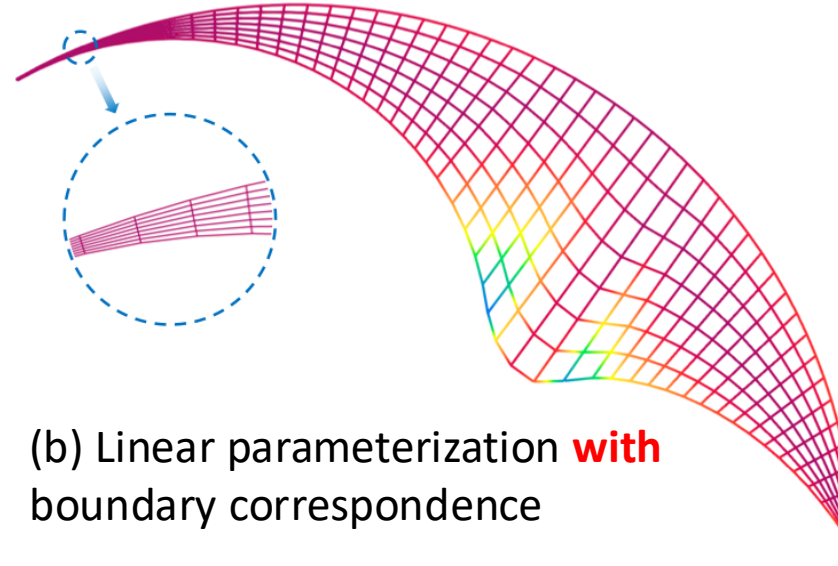
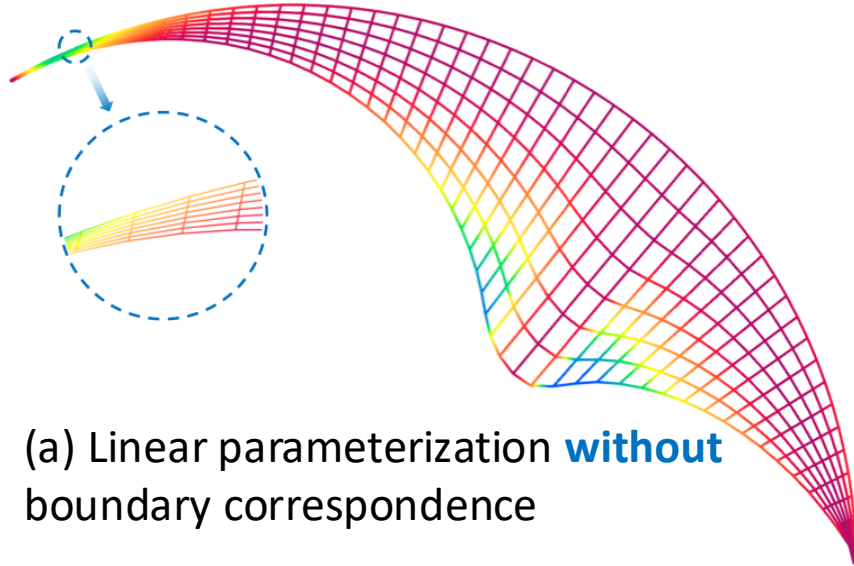
Deep learning²

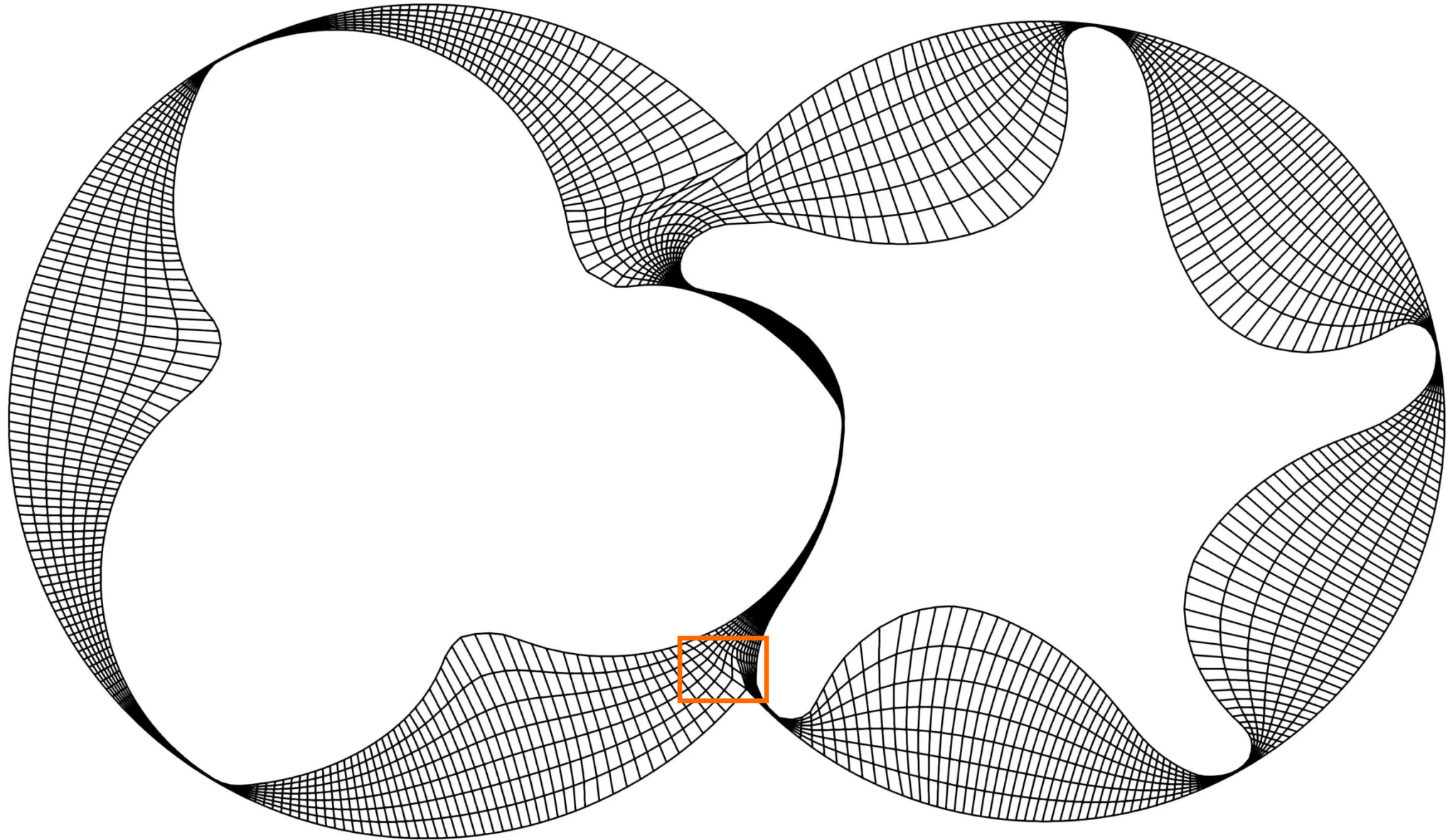


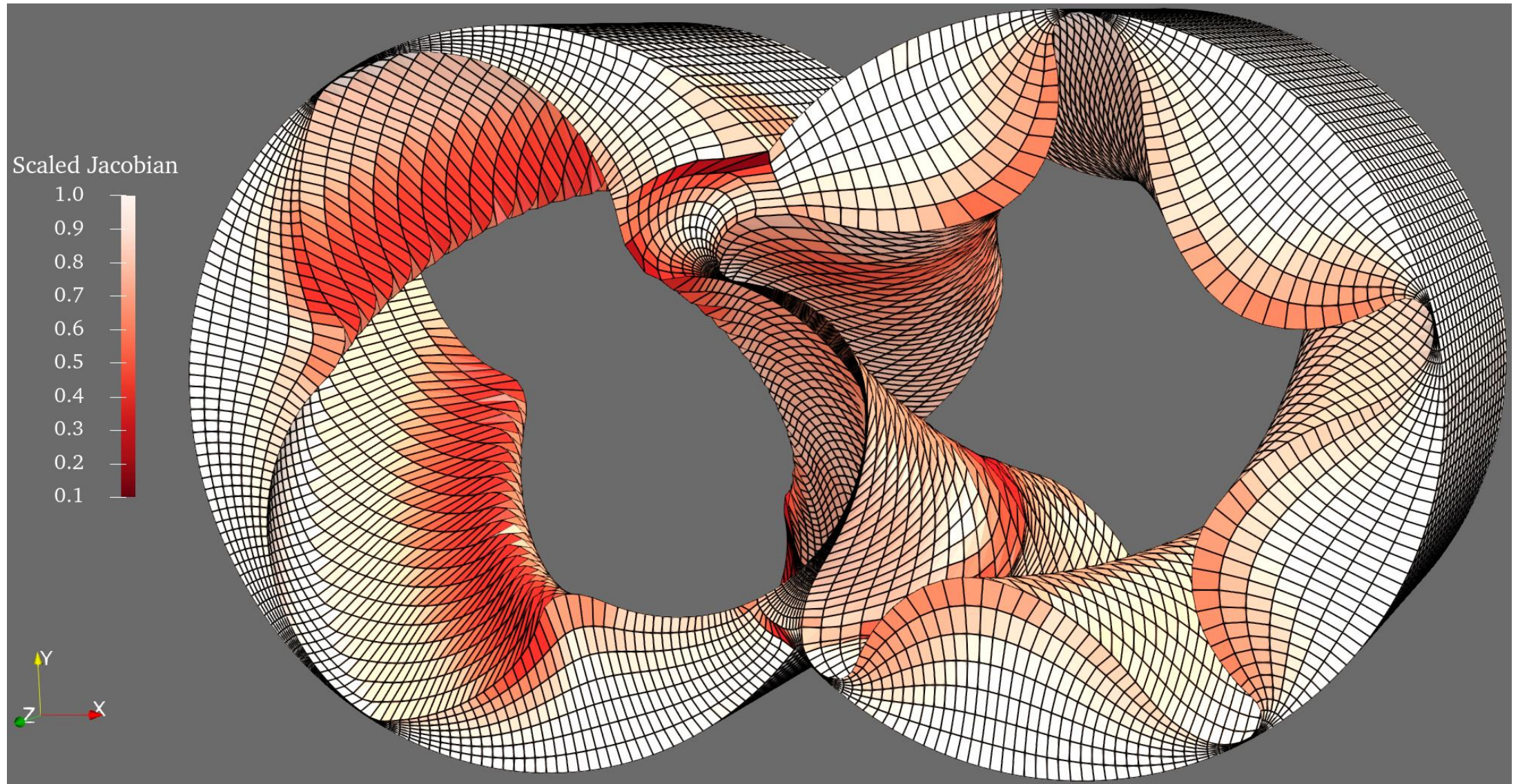
Input

Ours³

1. Zheng, Y., Pan, M., & Chen, F. (2019). Boundary correspondence of planar domains for isogeometric analysis based on optimal mass transport. *Computer-Aided Design*, 114, 28-36.
2. Zhan, Z., Zheng, Y., Wang, W., & Chen, F. (2023). Boundary Correspondence for Iso-Geometric Analysis Based on Deep Learning. *Communications in Mathematics and Statistics*, 11(1), 131-150.
3. Ji, Y., Möller M., Yu Y., & C. Zhu. (2024). Boundary parameter matching for isogeometric analysis using Schwarz-Christoffel mapping. *Engineering with Computers*, 1-19.



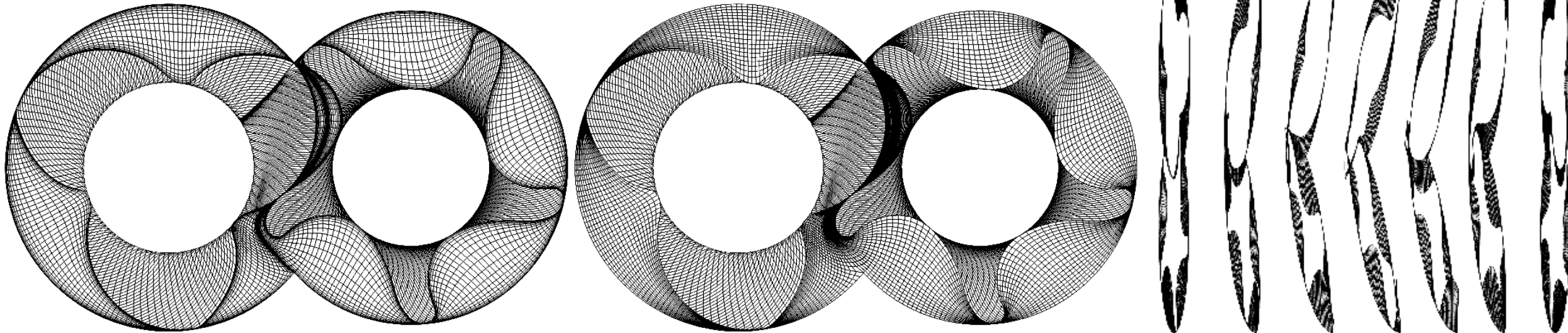
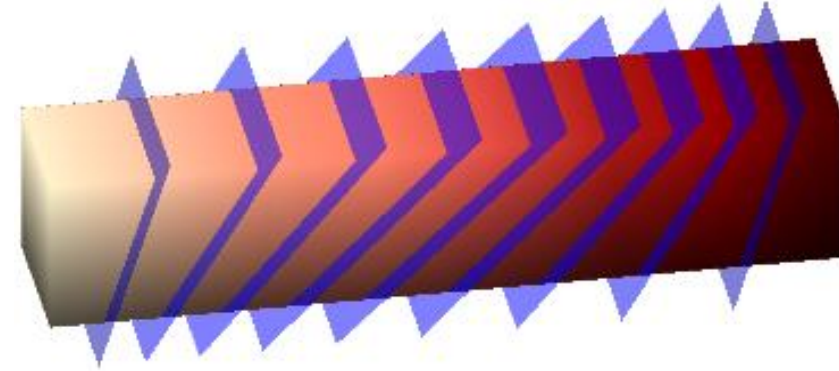


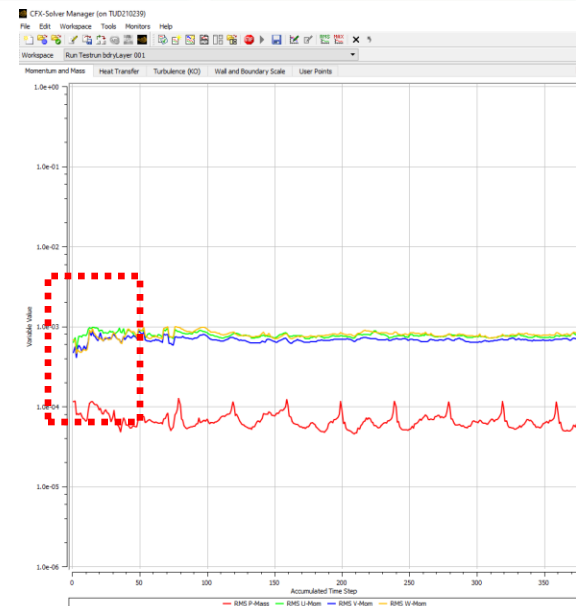
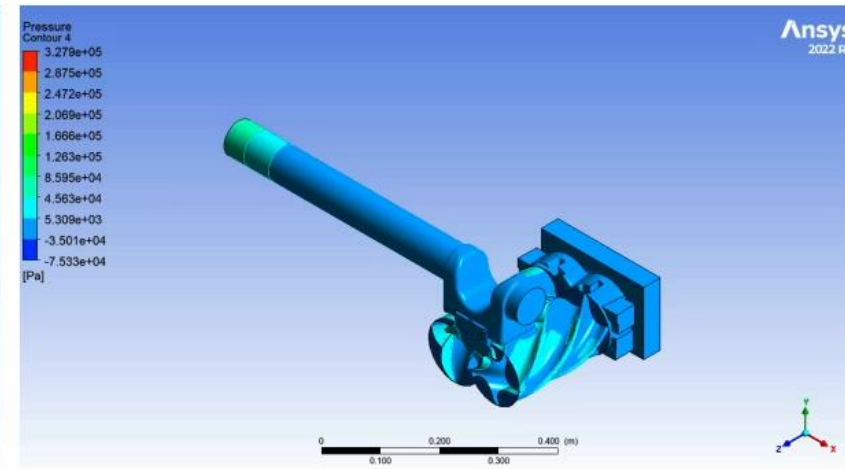
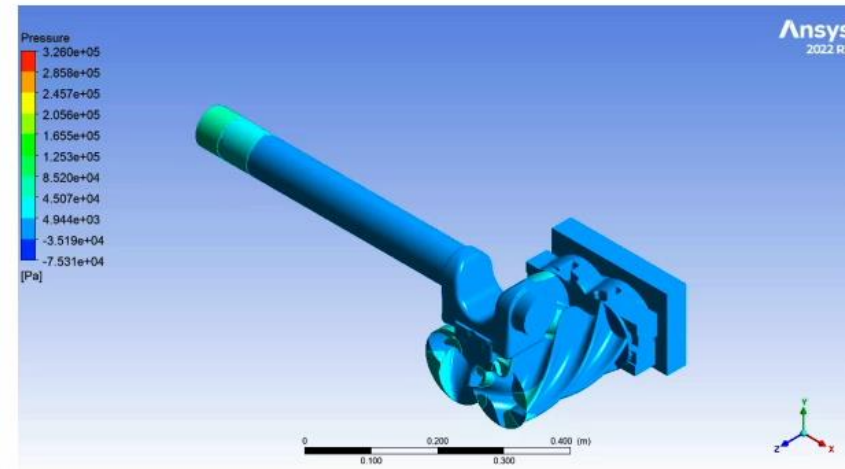
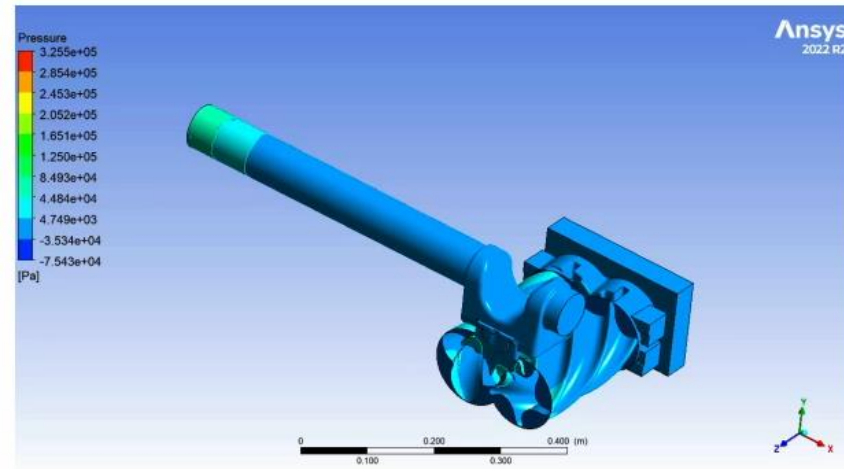


➤ Boundary layer mesh

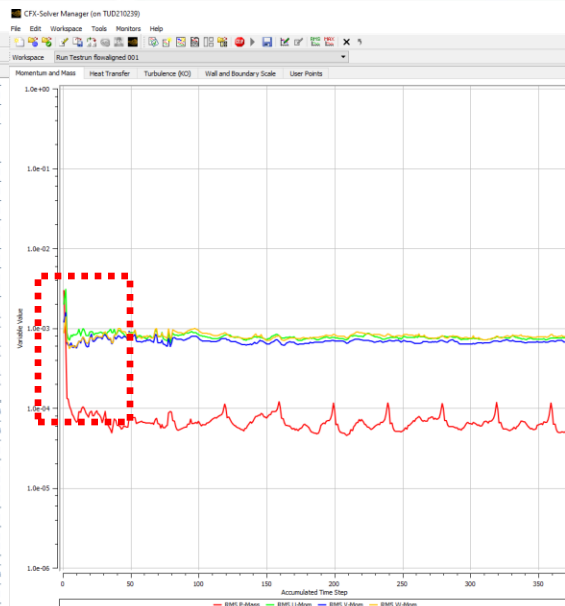
$$\begin{cases} \xi = \hat{\xi} \\ \eta = \frac{\tanh(\alpha(2\hat{\eta} - 1))}{2 \tanh(\hat{\eta})} + \frac{1}{2} \end{cases}$$

➤ Flow-aligned discretization





Equation	Rate	RMS Res	Max Res	Linear Solution
U-Mom	1.02	4.1E-03	9.4E-02	3.1E-02 OK
V-Mom	1.02	3.9E-03	3.9E-01	7.0E-02 OK
W-Mom	0.99	4.8E-03	2.0E-01	6.7E-02 OK
P-Mass	0.88	6.7E-04	1.2E-01	43.6 9.3E-02 OK
H-Energy	0.97	8.0E-03	4.8E-02	5.8 3.1E-04 OK
K-TurbKE	1.02	3.4E-03	5.2E-01	5.8 1.4E-03 OK
O-TurbFreq	1.00	6.4E-03	5.9E-01	5.5 1.6E-04 OK



Equation	Rate	RMS Res	Max Res	Linear Solution
U-Mom	0.64	1.9E-03	9.4E-02	1.4E-01 OK
V-Mom	0.62	1.0E-03	3.9E-02	2.1E-01 OK
W-Mom	0.61	1.3E-03	6.9E-02	1.5E-01 OK
P-Mass	0.58	1.1E-04	1.4E-02	43.6 4.0E-01 OK
H-Energy	0.59	1.3E-03	8.3E-02	5.8 1.9E-03 OK
K-TurbKE	0.73	1.3E-03	6.1E-02	5.8 1.9E-03 OK
O-TurbFreq	0.68	9.0E-04	6.7E-02	5.5 1.7E-04 OK

SCORG™

Boundary layer mesh

Flow-aligned mesh

C:\SCORG\ye-examples\N35 - [SCORG v2024]

File Edit Run View Units CAD Help

Machine Type: Screw Compressor

Inputs | Units | Properties

- N35
 - Profile
 - Geometry
 - Thermodynamics
 - Grids**
 - Control Switches
 - Rotor Mesh Size
 - Distribution Parameters
 - Meshing Parameters
 - Design

Control Switches

Rack Generation	Off
Rack Refinement Points	500
Boundary Generation	Off
Fluid Rotor Grid	On
Solid Rotor Grid	Off
Inlet Port Grid	Off
Outlet Port Grid	Off
Preprocessor Input File	Off
Vertex Files Start Number	1
Vertex Files End Number	40

Rotor Mesh Size

Circumferential Divisions Main Rotor	0
Circumferential Divisions Gate Rotor	40
Radial Divisions	7
Angular Divisions	40
Axial Divisions	0
Interlobe Divisions	50

Distribution Parameters

Type of Distribution	Casing to Rotor SplineMes
K Main	Rotor to Casing
K Gate	Casing to Rotor Nonconformal (Casing to Rotor Conformal)
Rack Smoothing Factor	Casing to Rotor SplineMesh
Project on Main profile	

Meshing Parameters

Mesh Orthogonality and Smoothing Control	
Relaxation Factor (0 - 1)	1
Tolerance Factor (1 - 100)	100
Inflation Layer Control	
Radial Bias Factor (0 - 1)	0.5

Distribution of computational points on the rotor boundary

User Profile | Geometry Inputs | Thermodynamic Inputs | **Rotor Grid**

Distribution: 41 | Mesh: 41 | Quality Criteria: None | Show Lines | Show Rotors | Quality Report

SplineMesh

Output Window

```

cell statistics          overall number of cells          0
. Rotor fluid           0          . Inlet port           0
. Rotor solid           0          . Outlet port          0

Start: 15: 4: 3   End: 15: 4: 5   Running time: 0h: 0m: 2s = 3 sec
4/ 1/2025...
SCORG - screw compressor Rotor Geometry grid generator - Ver. 2024
                    
```

Machine Type: Screw Compressor

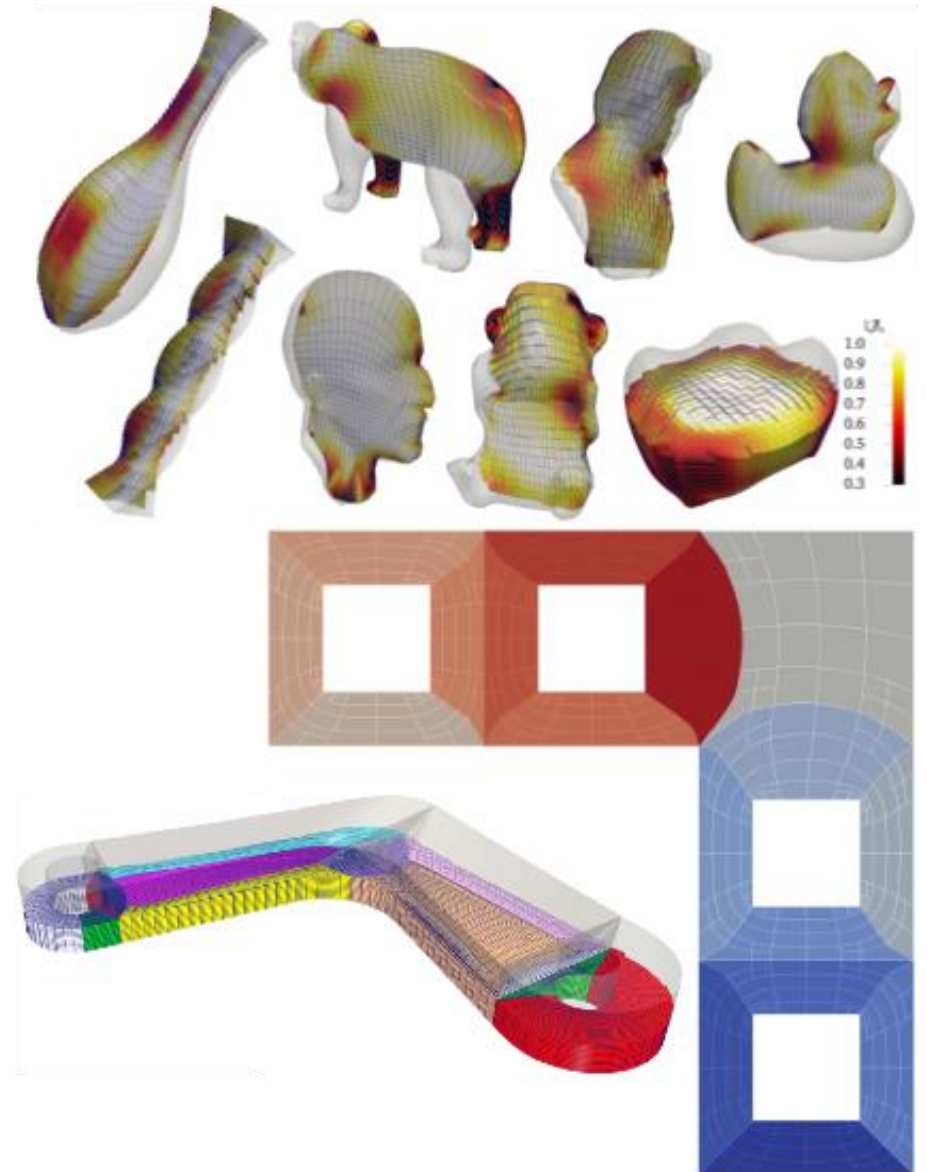
- N35
 - Design
 - Grid
 - Profile
 - Thermodynamics
 - N35.spf
 - Settings.xml

Design Data	Main Rotor	Gate Rotor	Unit
Axis Distan...	0.09...	0.09...	m
Outer Diam...	0.12...	0.12...	m
Pitch Diam...	0.06...	0.11...	m
Inner Diam...	0.06...	0.06...	m
Rotor Length	0.21...	0.21...	m
Lead	0.25...	0.41...	m
Wrap Angle	306...	183...	Deg
Lead Angle	48.778	48.778	Deg
Helix Angle	41.222	41.222	Deg

Outline

1. Background and motivation
2. Overview of the algorithms
3. Applications
4. Conclusions and outlook

- **G+Smo offers** three major classes of **parameterization methods**;
- Demonstrates **improved robustness and efficiency** over existing methods;
- Applicability in both **academic** and **real-world industry scenarios**;
- Future Work:
 - Integrating newly developed and ongoing methods into G+Smo for broader usability;
 - User-friendly **graphical user interface**.



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Eindhoven, The Netherlands

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TU/e

EINDHOVEN
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Many thanks for your attention!

Q&A

If you are interested in my research, please feel free to contact me! ;-)

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