Analysis-suitable parameterization for isogeometric analysis

Ye Ji (纪野)

Department of Applied Mathematics

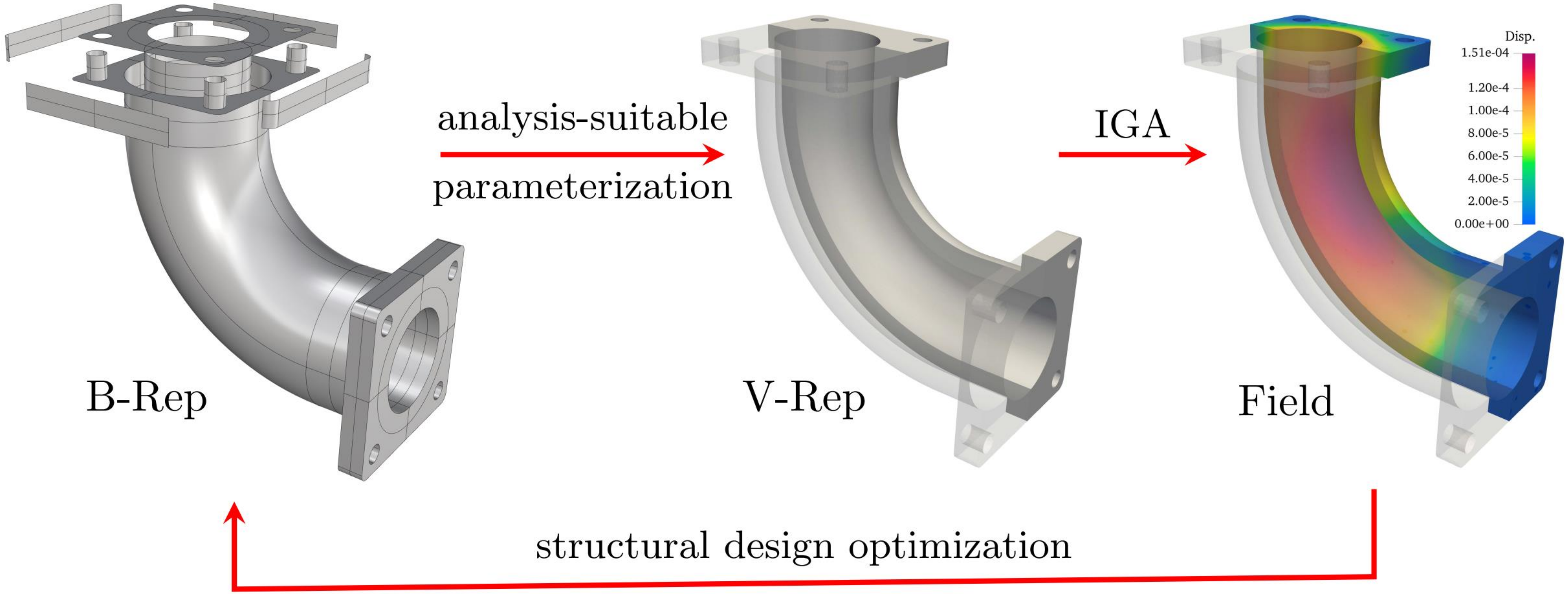
Delft University of Technology, The Netherlands

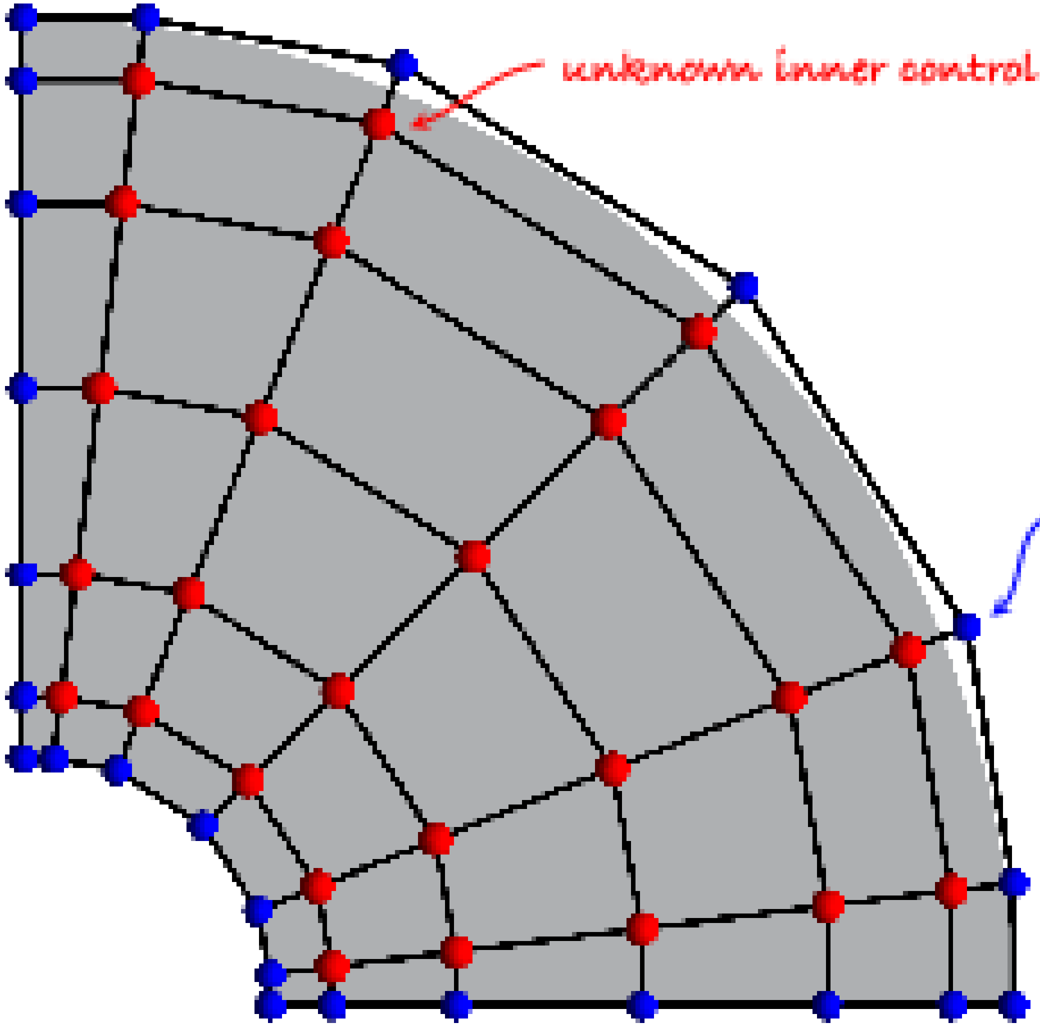
Outline

1. Background and motivation
2. Overview of the algorithms
3. Applications
4. Conclusions and outlook

Outline

- CAD models are usually represented by **boundary representation** (B-Rep);
- However, IGA requires an internal **spline-based parameterization** (V-Rep).





$$\boldsymbol{x}(\boldsymbol{\xi}) = \underbrace{\sum_{i \in \mathcal{I}_I} \mathbf{P}_i R_i(\boldsymbol{\xi})}_{\text{unknown}} + \underbrace{\sum_{j \in \mathcal{I}_B} \mathbf{P}_j R_j(\boldsymbol{\xi})}_{\text{known}}.$$

GOAL: to construct **unknown inner control points $\mathbf{P}_i$** (or basis functions R_i) such that x ensures **bijectivity** and exhibits **optimal orthogonality** and **uniformity**.

- **Parameterization quality** significantly affects **downstream analysis!**

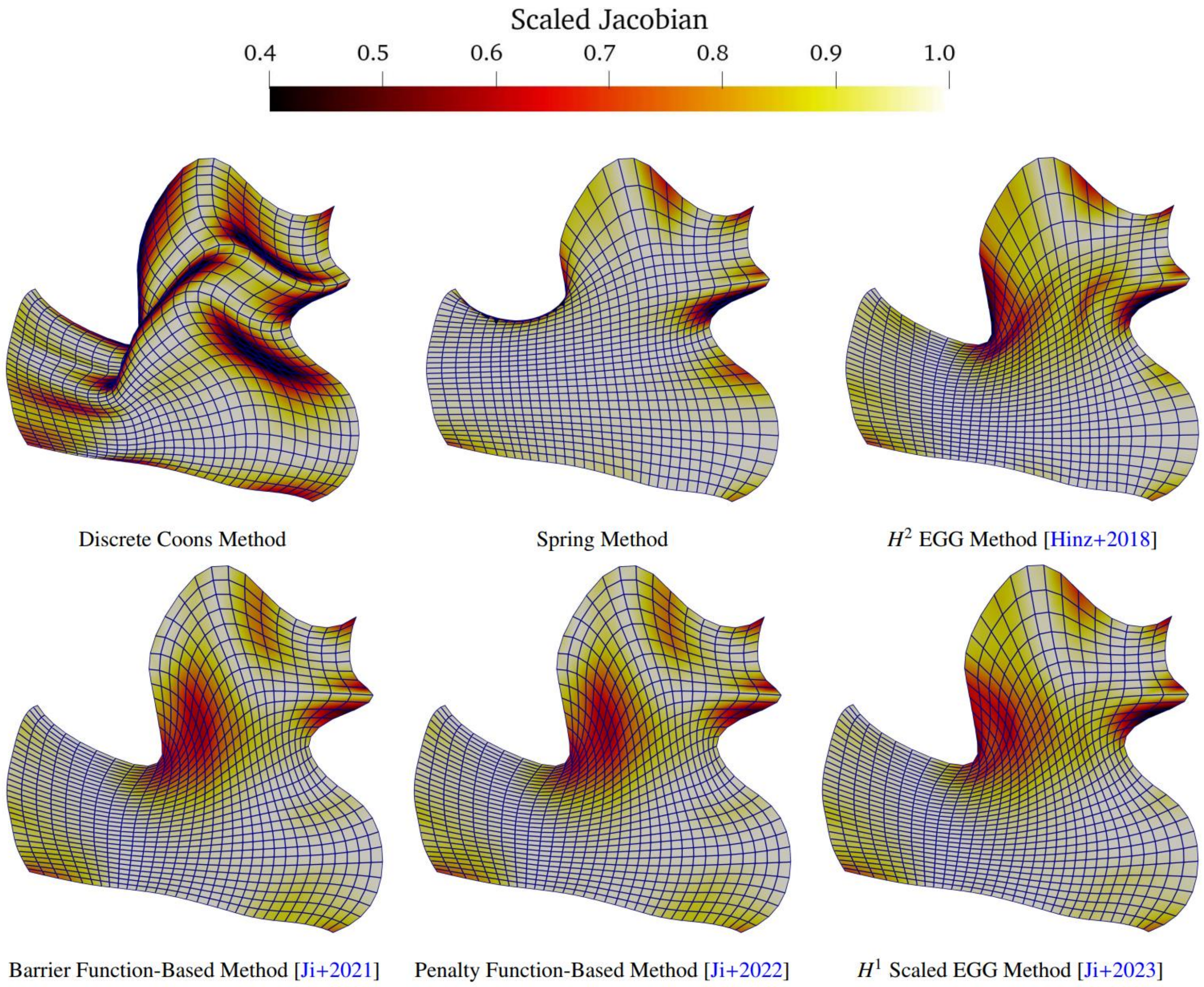
Outline

as_parameterization_example.cpp in G+Smo

Class	Brief Description
gsBarrierCore<d, T>	Core class for AS-parameterizations using various approaches.
gsBarrierPatch<d, T>	Data and geometry preprocessing for gsBarrierCore<d, T>.
gsHLBFGS<T>	Wrapper for the Hybrid Low-storage BFGS optimization solver.
AndersonAcceleration<T>	Anderson acceleration solver and its preconditioned variants.
preAAParam<T>	Parameters for the preconditioned AA solver.

- Example file: `as_parameterization_example.cpp;`
- Test inputs: filedata/breps
  - 2D case: `filedata/breps/2D/duck_BRep.xml`;
  - 3D case: `filedata/breps/3D/duck_BRep.xml`;
  - Multi-patch: `filedata/breps/other/TUDflame.xml`, **credits to Hugo Verhelst**.
- **Many available parameterization methods in G+Smo**.

Available methods in G+Smo



- Algebraic methods:
  - Coons patch
  - Spring patch
- Optimization-based methods:
  - Barrier-function-based [1]
  - Penalty-function-based [2]
- PDE-based methods:
  - Elliptic grid generation [3]
  - Improved EGG [4]

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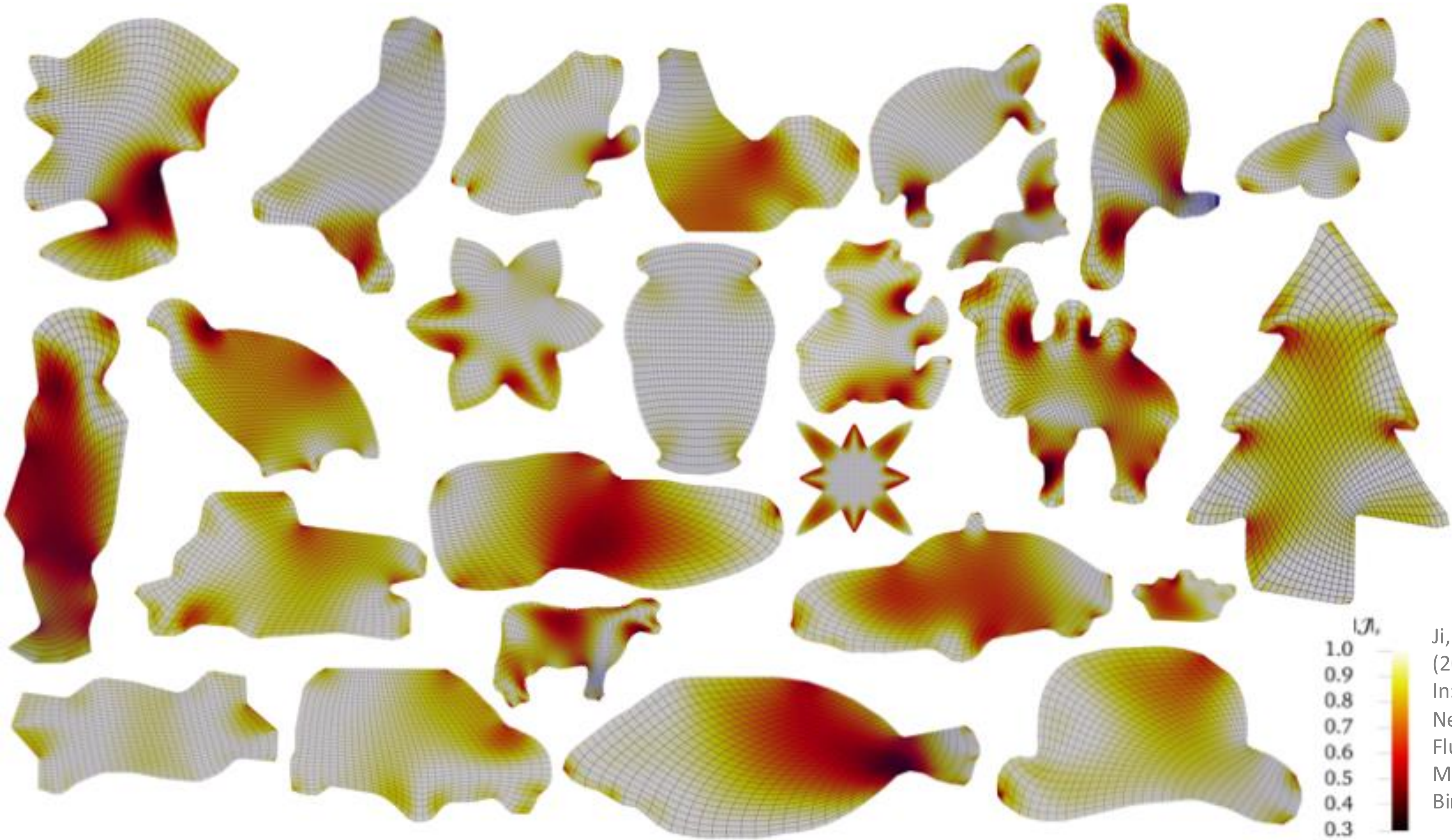
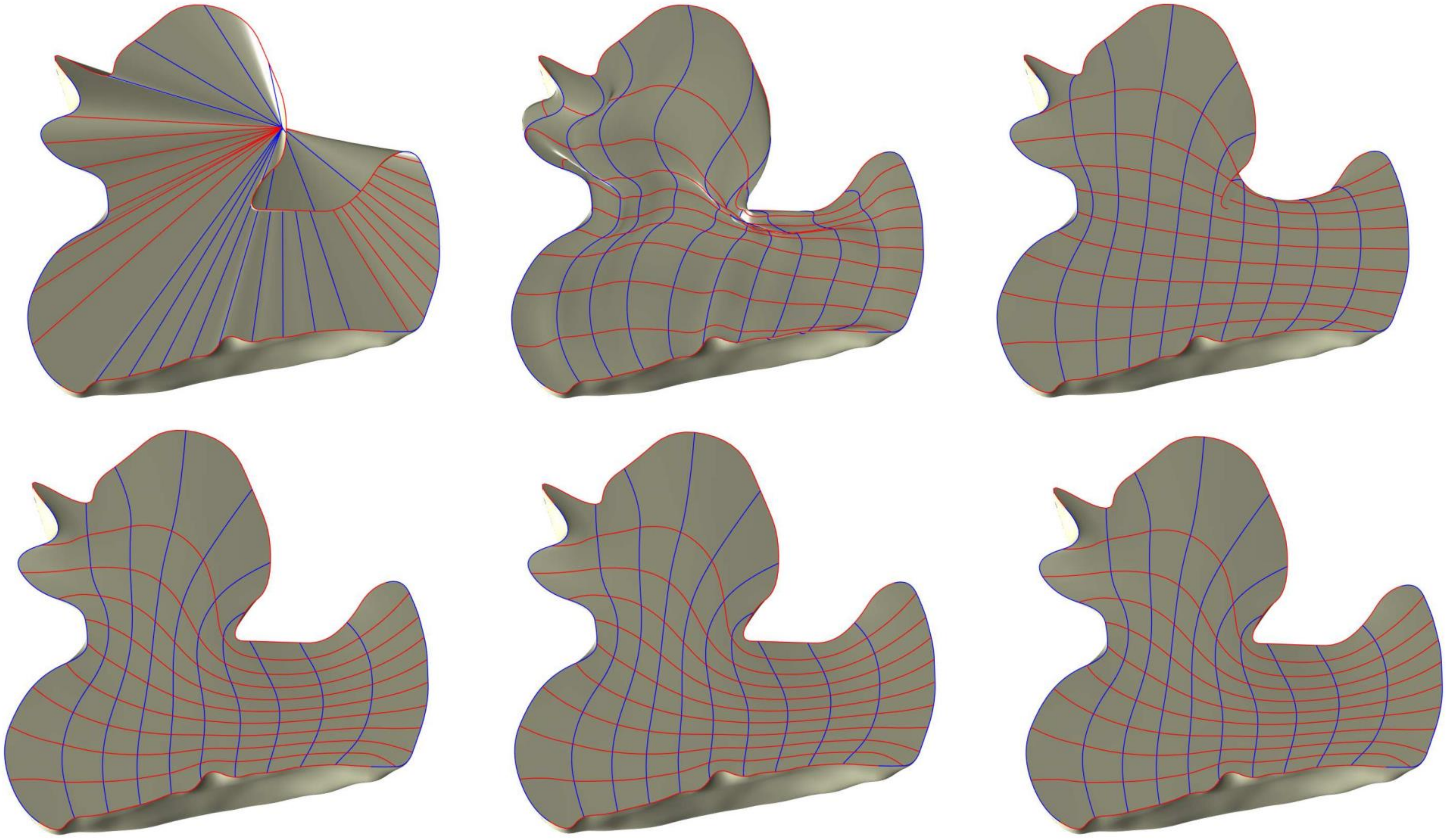
1. Ji, Y. et al. (2021). JCAM, *396*, 113615.
2. Ji, Y. et al. (2022). CAGD, 94, 102081.
3. Hinz, J. et al. (2018). CAGD, *65*, 48-75.
4. Ji, Y. et al. (2023). *CAGD*, *102*, 102191.

Planar Parameterization Test Dataset (977 models)



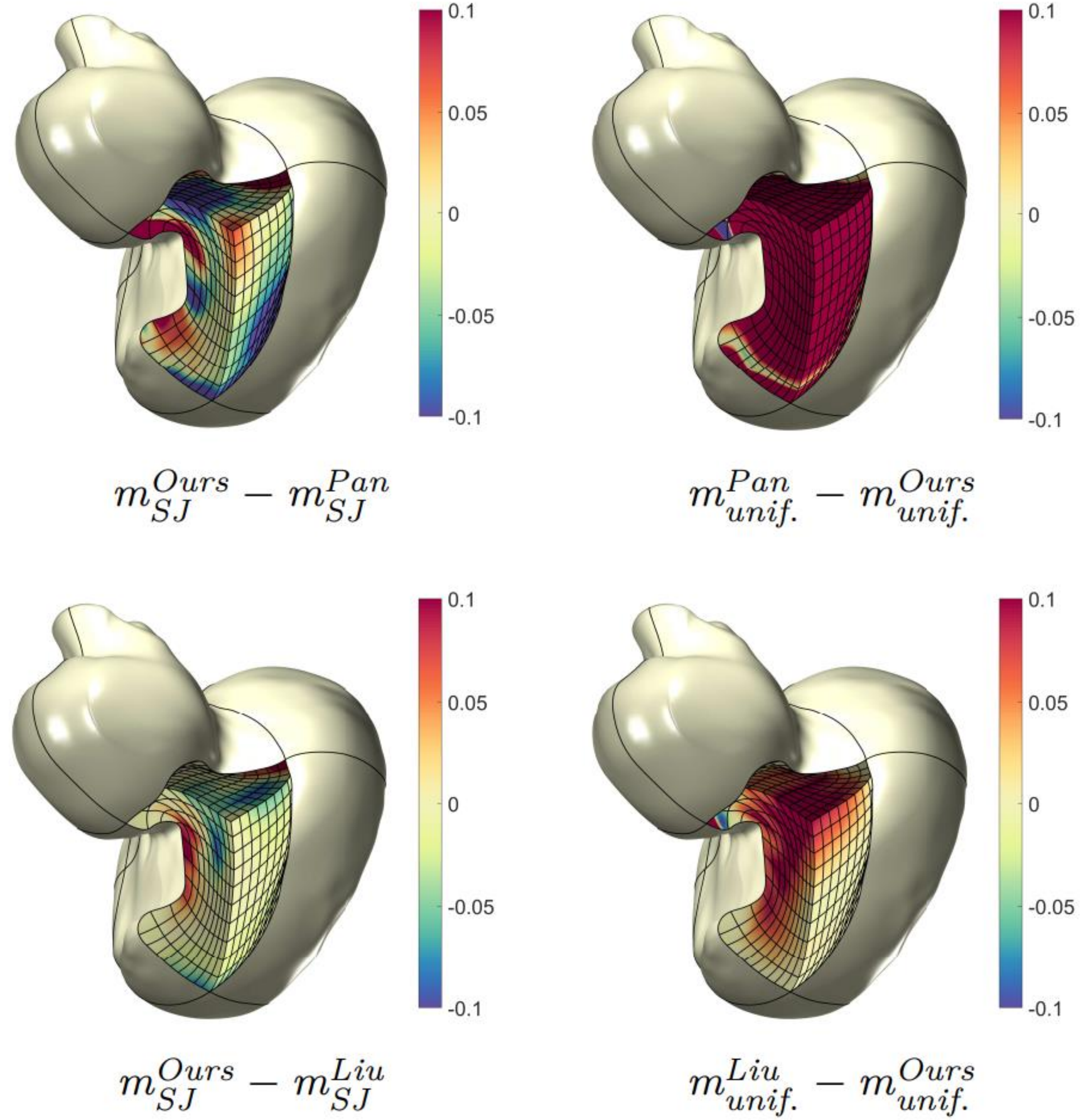
Ji, Y., Möller, M., Verhelst, H.M. (2024). Design Through Analysis. In: Bodnár, T., Galdi, G.P., Nečasová, Š. (eds) Fluids Under Control. Advances in Mathematical Fluid Mechanics. Birkhäuser, Cham.

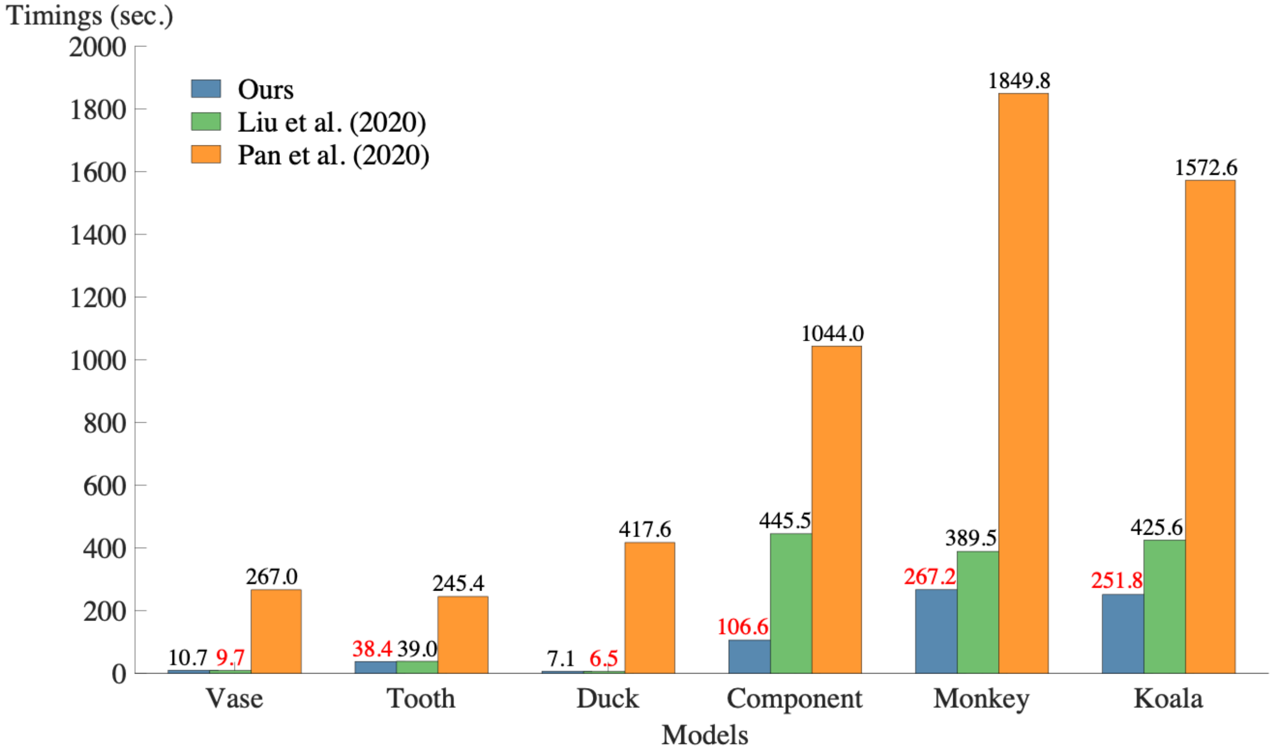
Strong similarity from different initializations



Same point　　Coons Patch　　Smoothness energy

Comparisons with existing methods

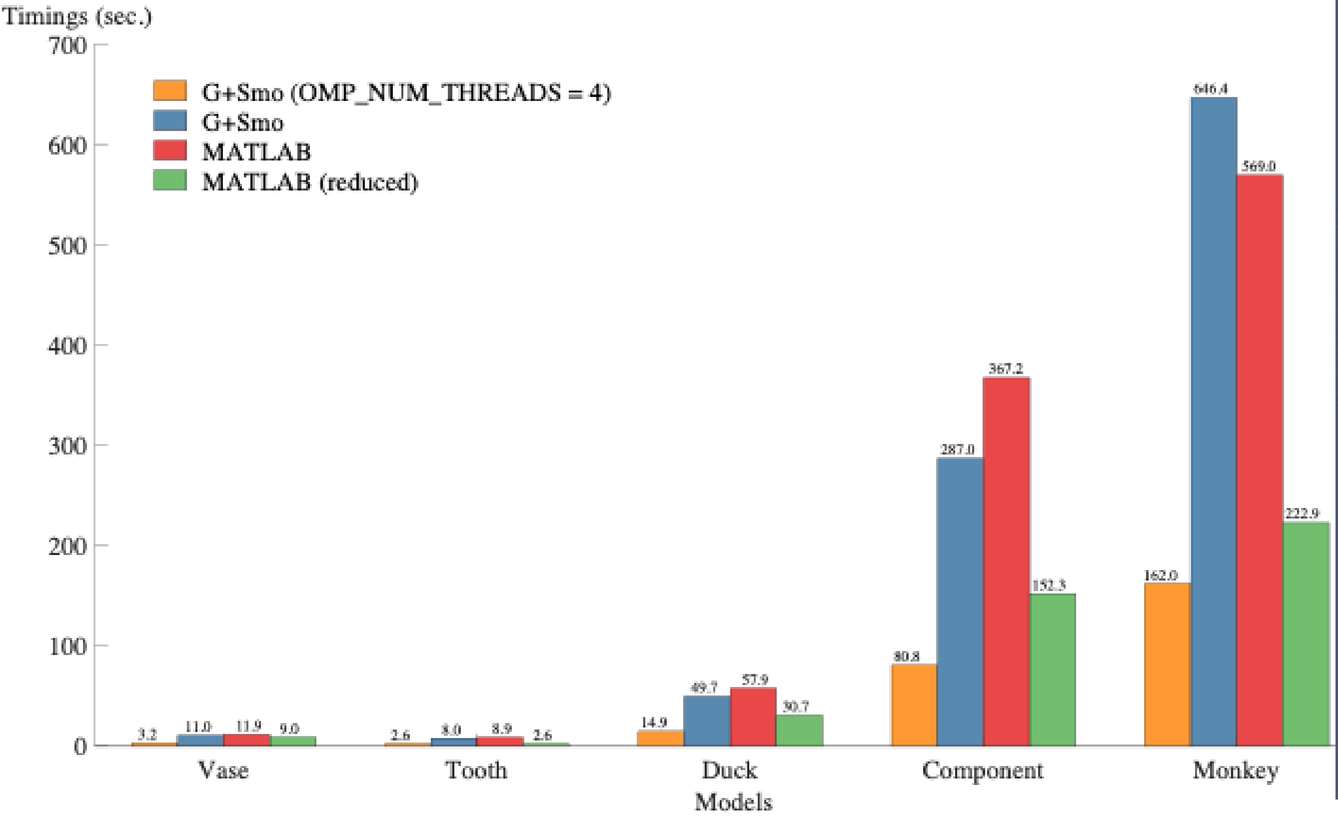


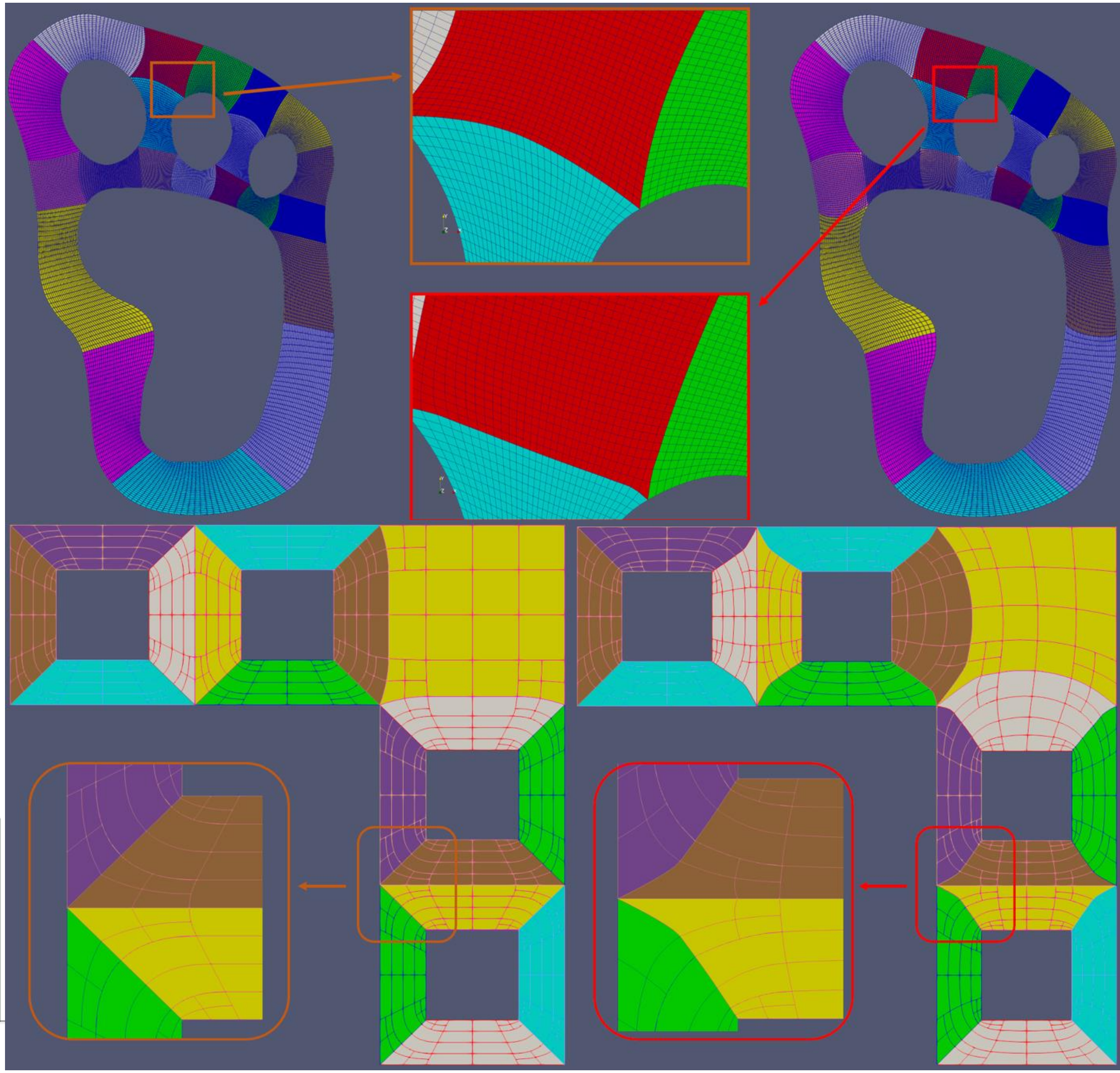


- **Positive values** indicate our method performs better;
- Efficiency comparison (MATLAB vs. C++ (Pan+ & Liu+):
  - Significantly **faster** than Pan et al. (2020);
  - Large-Scale Models: Outperforms Liu et al. (2020).

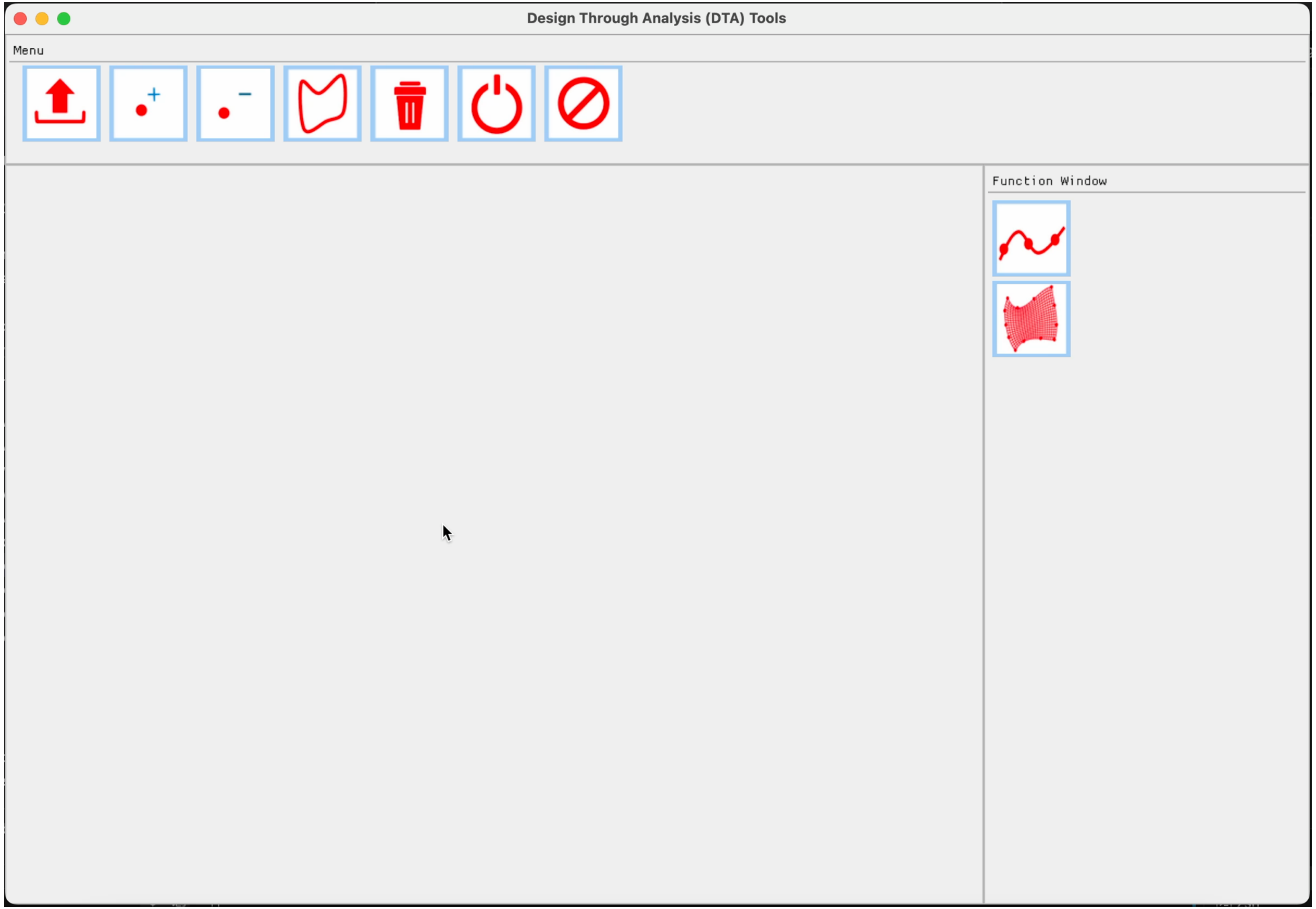
1. Pan, M., Chen, F., & Tong, W. (2020). CMAME, 359, 112769.
2. Liu, H., Yang, Y., Liu, Y., & Fu, X. M. (2020). CAGD, 79, 101853.

- In our released G+Smo implementation, **3-4x speed-up**;
- Suitable for **multi-patch** and **THB-spline parameterization**;

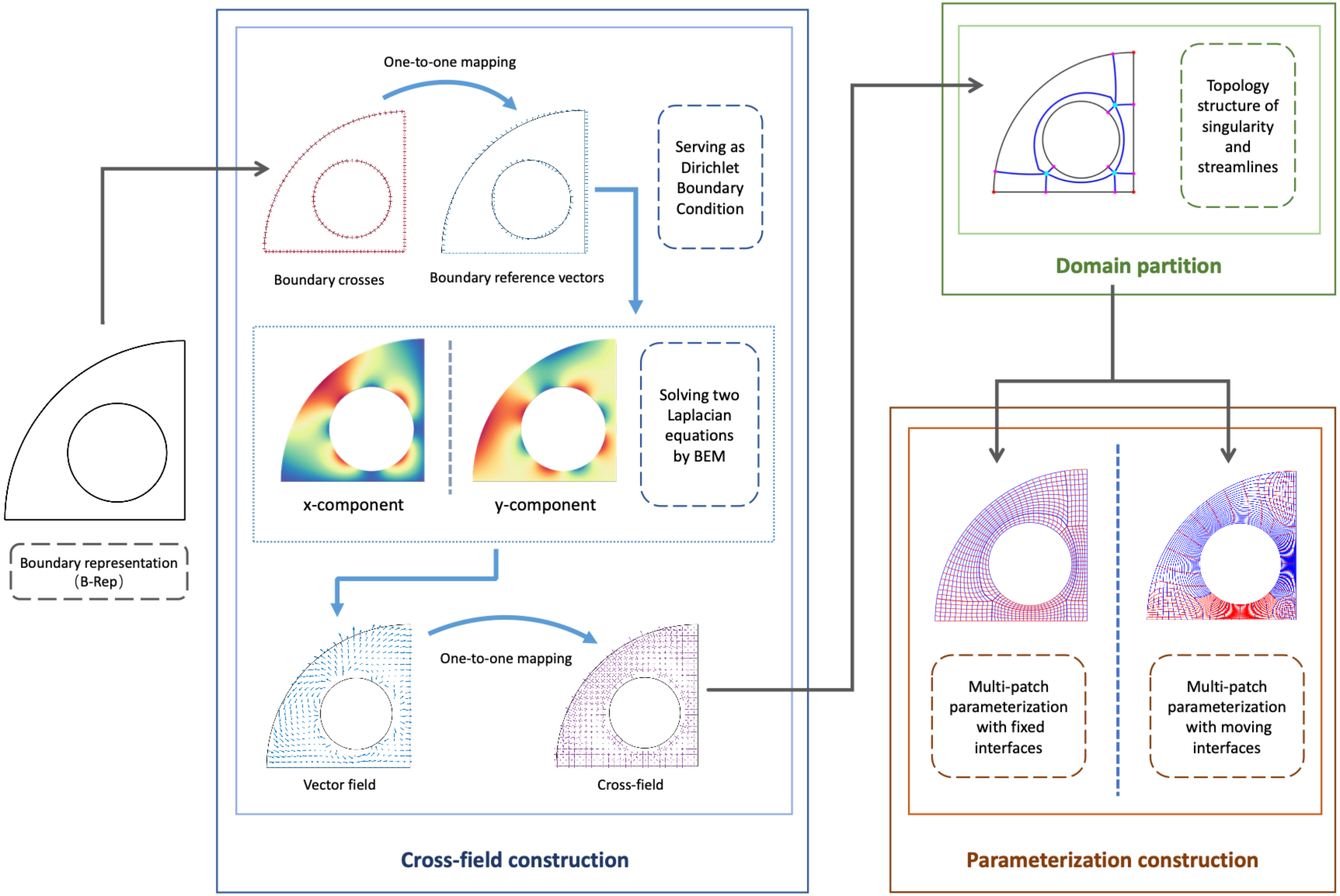


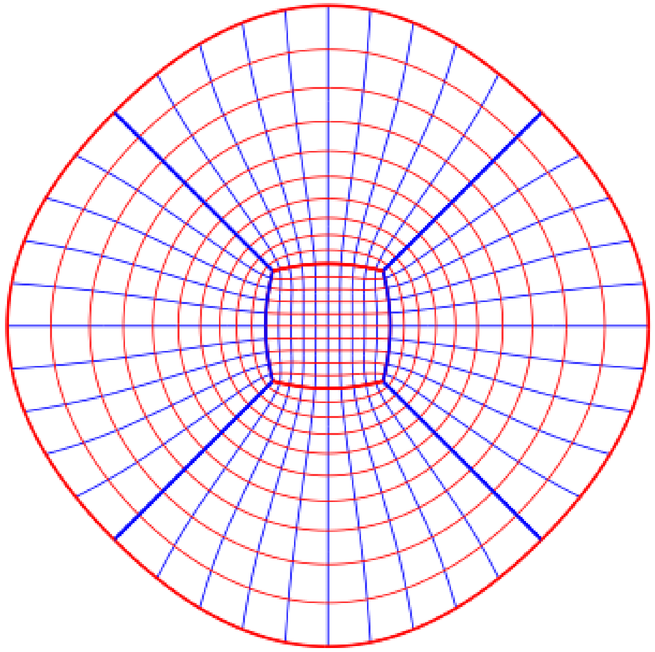


GUI: Design Through Analysis (DTA) Tools



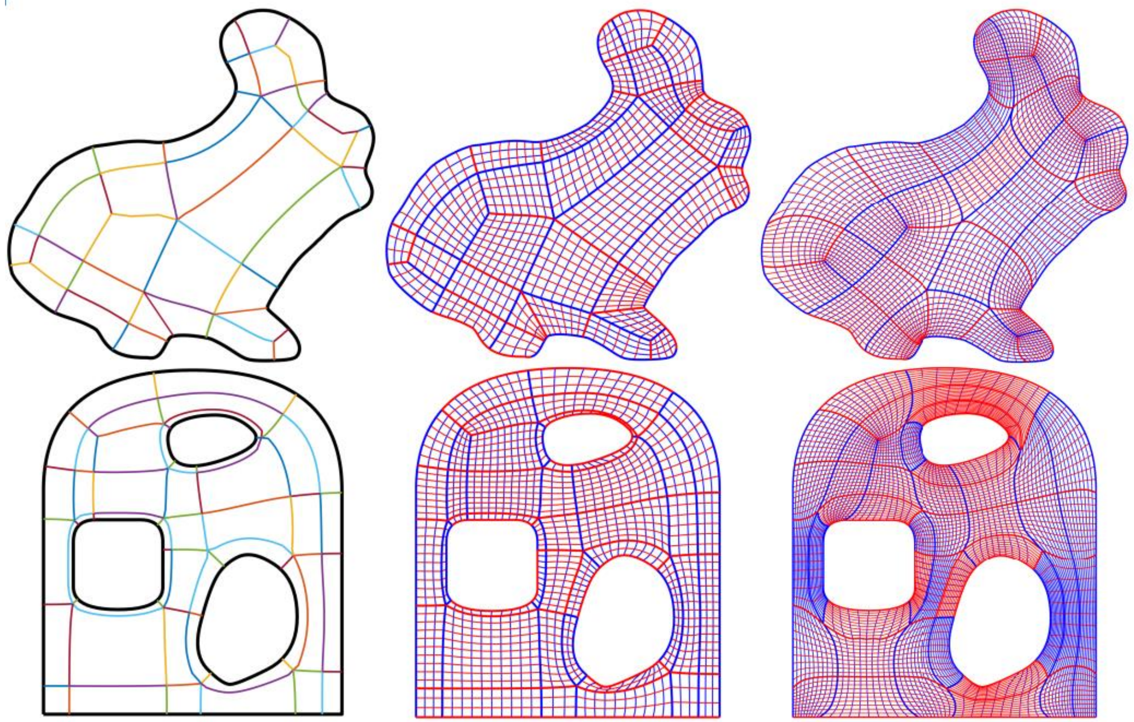
Multi-patch parameterization using cross-field



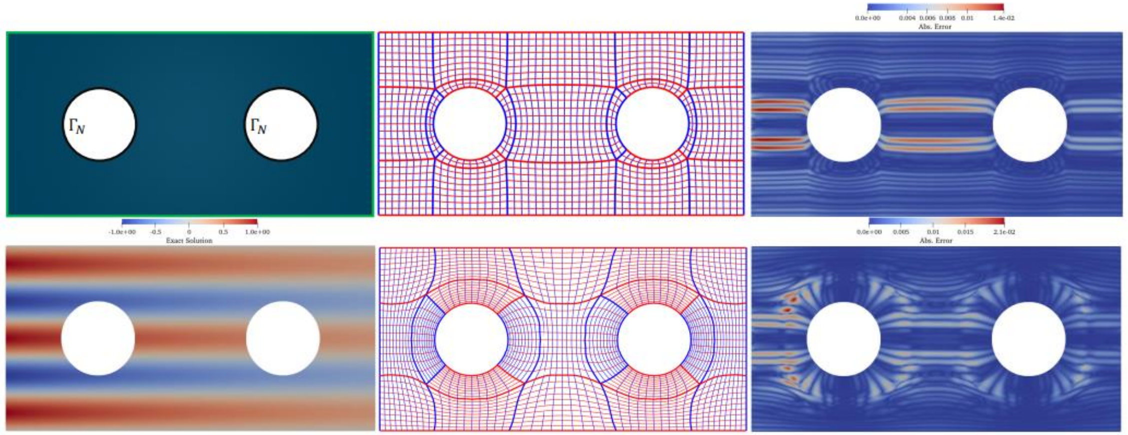


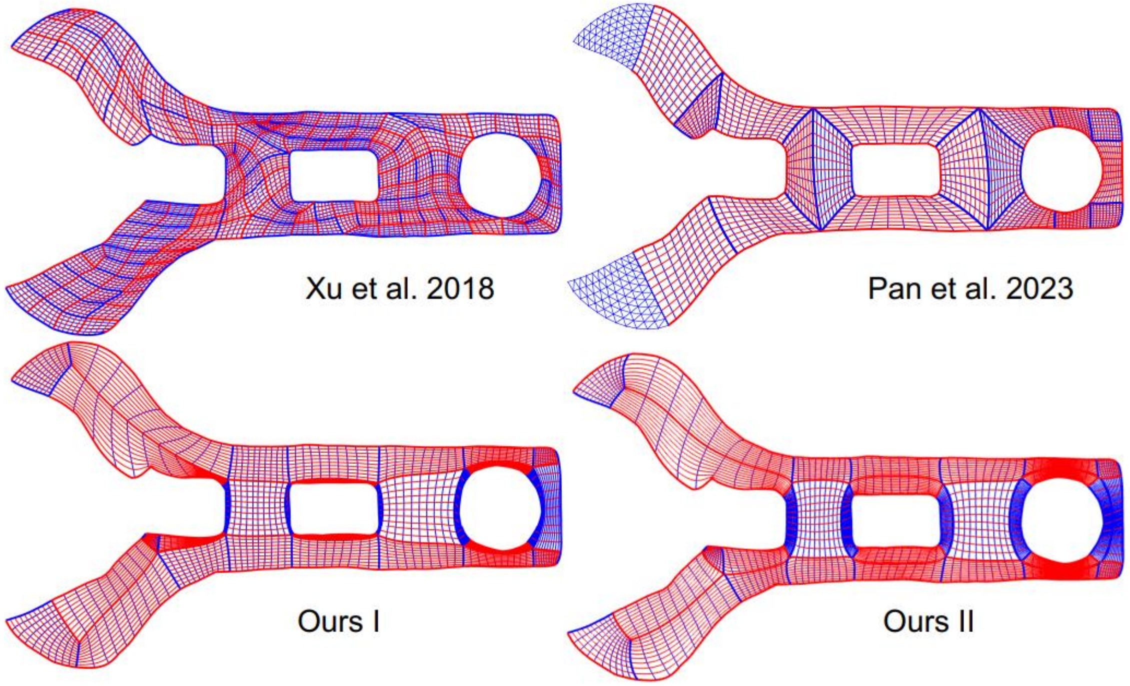
Zhang, Y., Ji, Y., & Zhu, C. G. (2024). Multi-patch parameterization method for isogeometric analysis using singular structure of cross-field. *Computers & Mathematics with Applications*, *162*, 61-78.

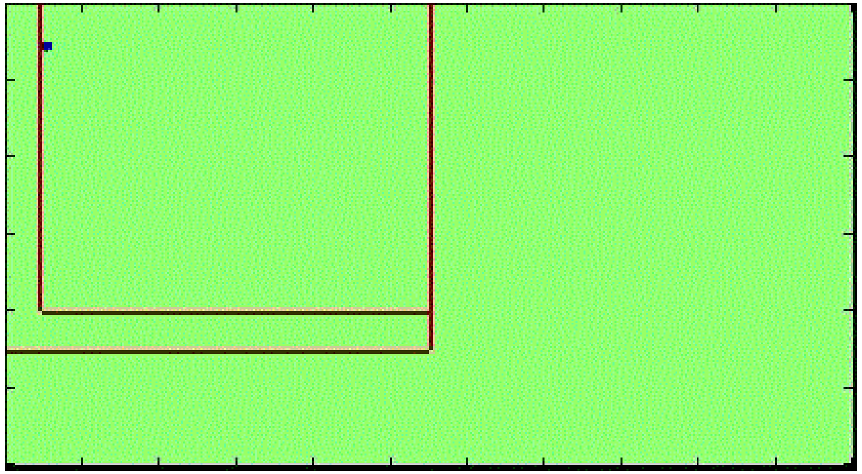
Multi-patch parameterization: Results and comparisons



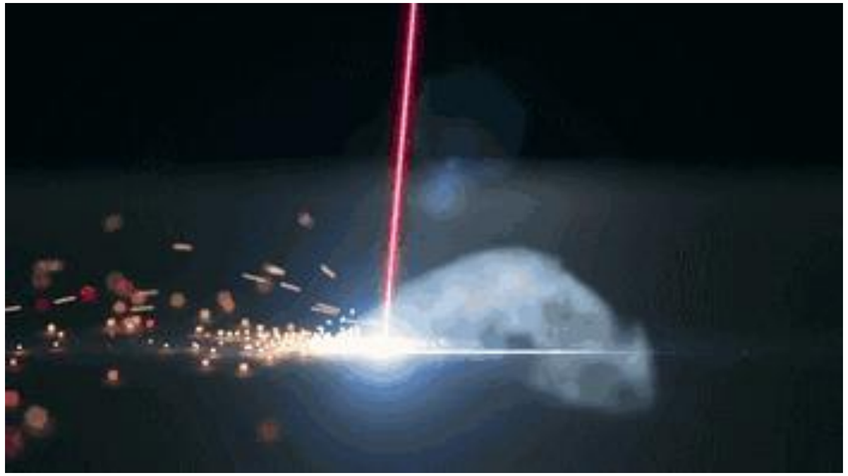
Model	#Patch	Method	$\lvert\mathcal{J}\rvert_s$ min.	$\lvert\mathcal{J}\rvert_s$ avg.	*unif.* min.	*unif.* avg.	*unif.* max.
rabbit	33	Coons	-0.8593	0.9628	0.7030	**0.9410**	1.0524
		fixed-I	**0.2204**	**0.9504**	0.6103	0.9544	**0.9982**
		moving-I	0.02918	0.9283	**0.0000**	0.9550	1.0000
3 holes	46	Coons	-0.5492	0.9710	0.8008	0.9573	1.0958
		fixed-I	**0.1545**	**0.9716**	0.8007	0.9573	0.9978
		moving-I	0.1461	0.9361	**0.6791**	**0.9571**	**0.9968**



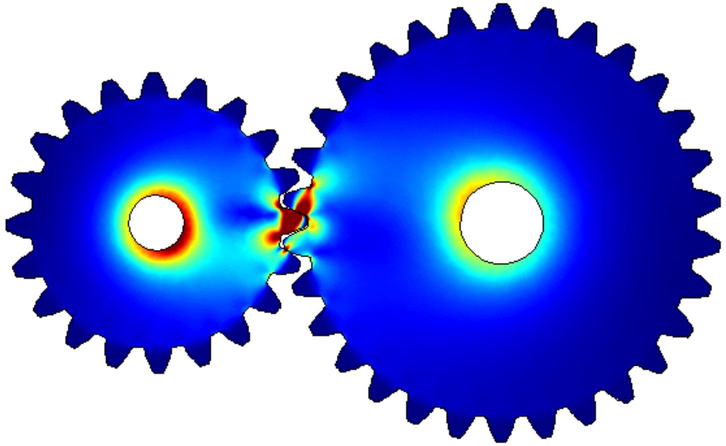




Wave propagation



Laser printing

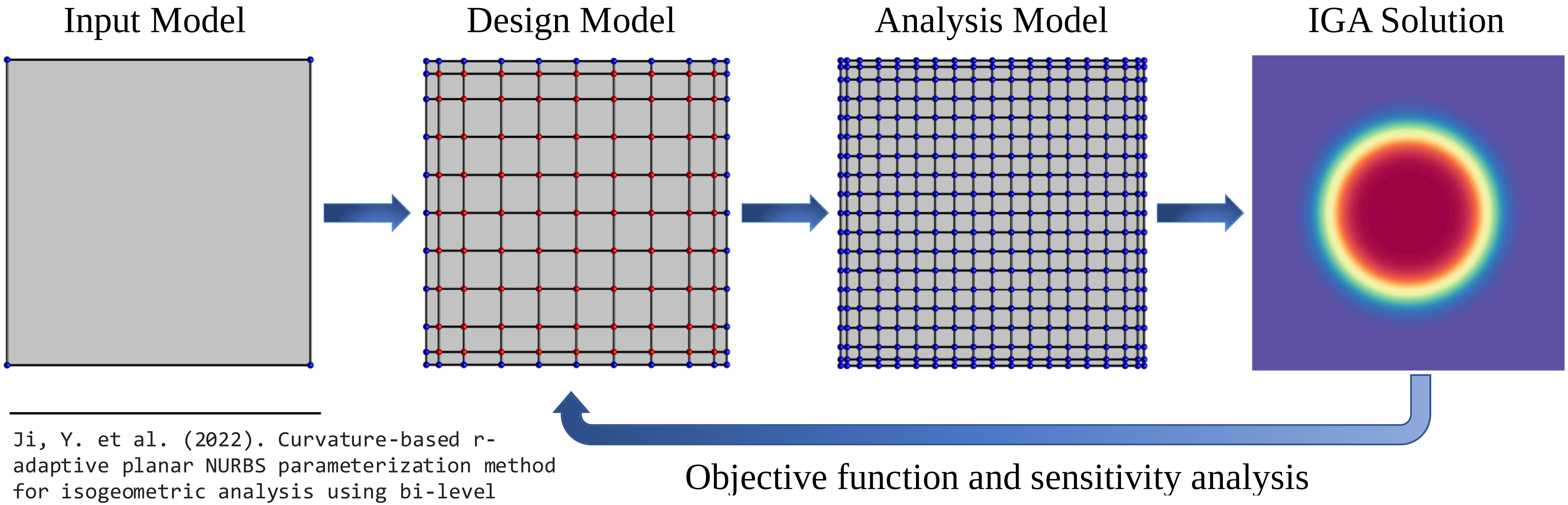


Stress distribution

- **Localized and anisotropic features** extensively exist in physical phenomena;
- **Isotropic parameterizations are not efficient** for such problems;
- Anisotropic parameterizations (r-adaptivity):
  - Enhance per-DOF accuracy while keeping constant total DOFs.
  - Keep the sparse pattern of mass matrix and stiffness matrix.

Anisotropic parameterizations are often solution-dependent:

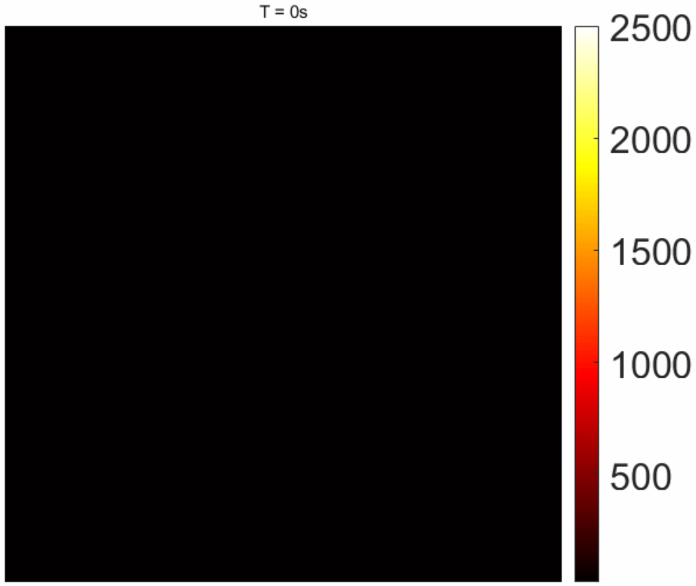
- Need **good numerical solution accuracy** to drive parameterization;
- Adjust as few control points as possible **for high efficiency**;
- **Bi-level strategy:** a coarse level (design model) to update the parameterization for efficiency and a fine level (analysis model) to perform analysis for accuracy.

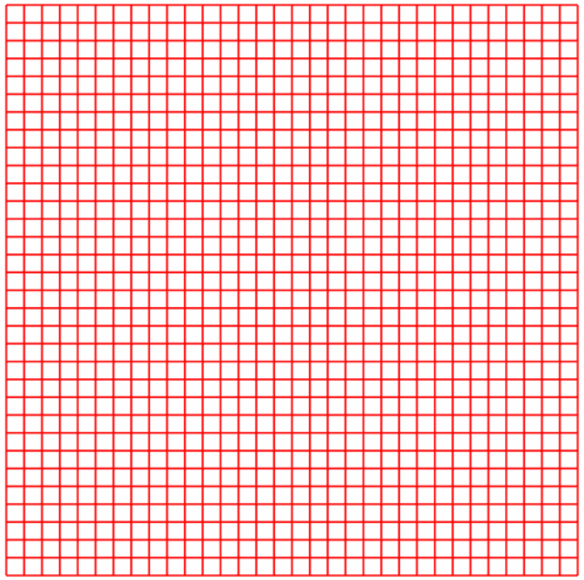


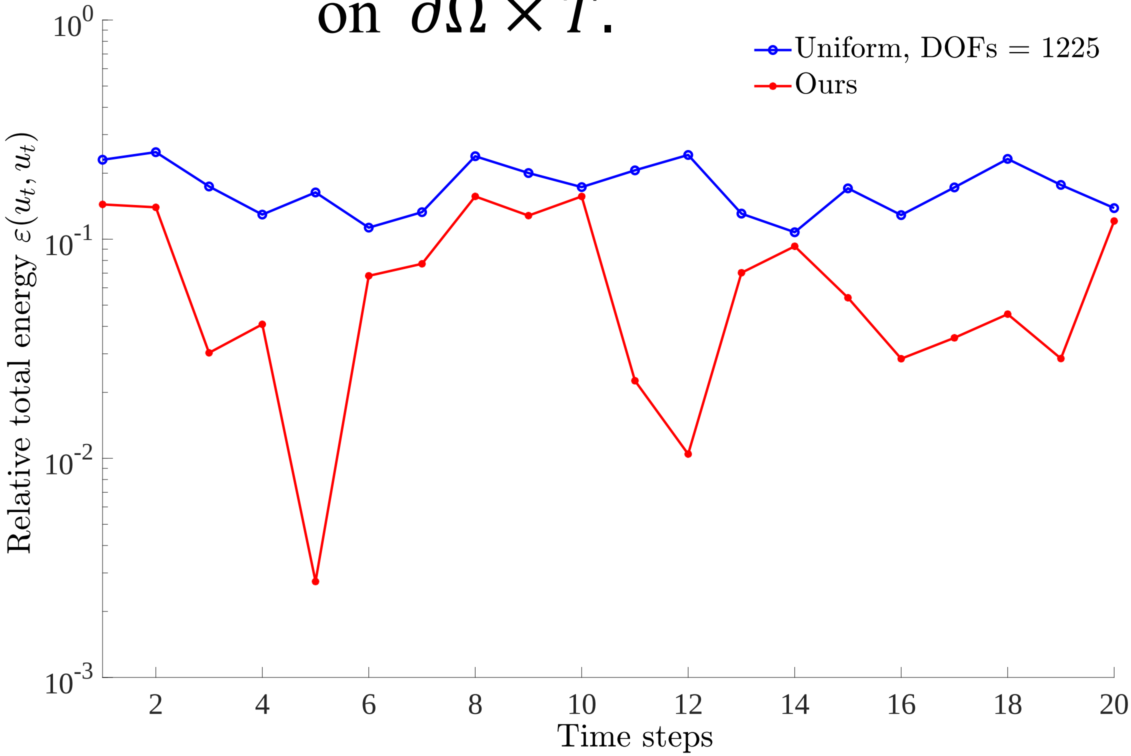
Ji, Y. et al. (2022). Curvature-based r-adaptive planar NURBS parameterization method for isogeometric analysis using bi-level approach. *Computer-Aided Design, 150,* 103305.

- Consider a two-dimensional linear heat transfer problem with a moving Gaussian heat source:

$$\begin{cases} C_p \rho \dfrac{\partial u(\boldsymbol{x},t)}{\partial t} - \nabla \cdot \big(\kappa \nabla u(\boldsymbol{x},t)\big) = f(\boldsymbol{x},t), & \text{in } \Omega \times T, \\ u(\boldsymbol{x},t) = u_0, & \text{in } \Omega, \\ \kappa \nabla u(\boldsymbol{x},t) = 0, & \text{on } \partial\Omega \times T. \end{cases}$$







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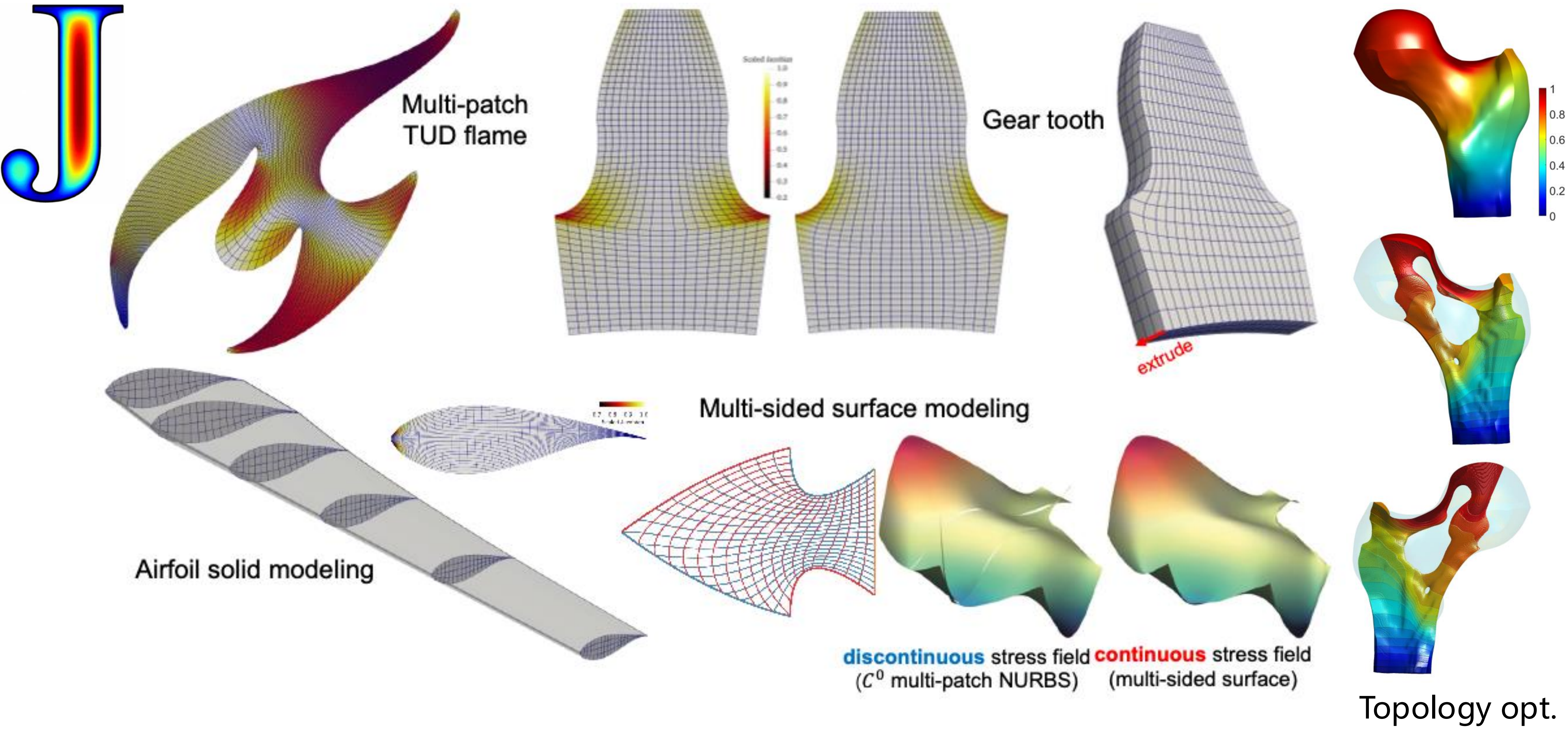
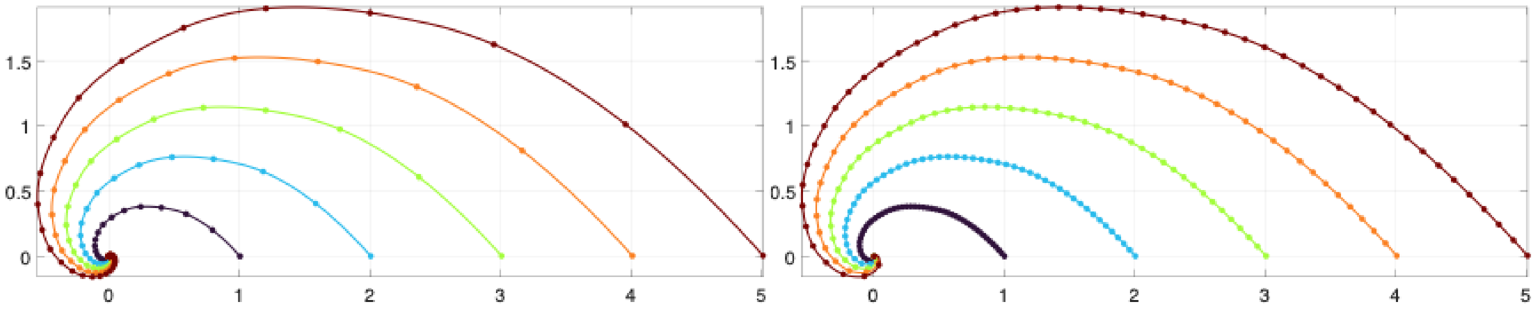
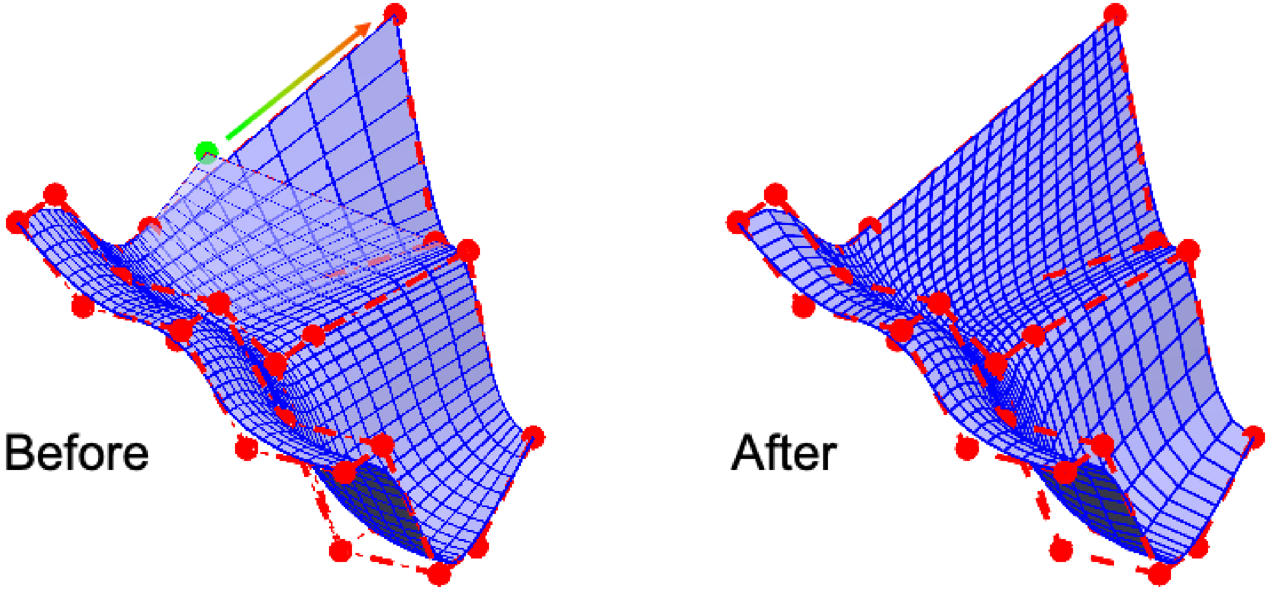
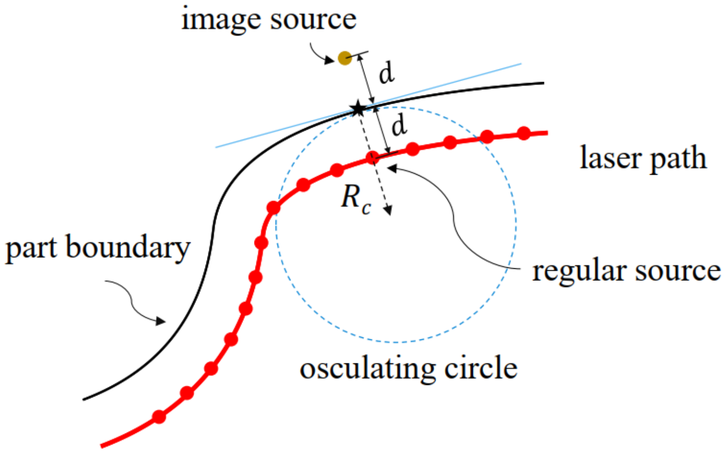
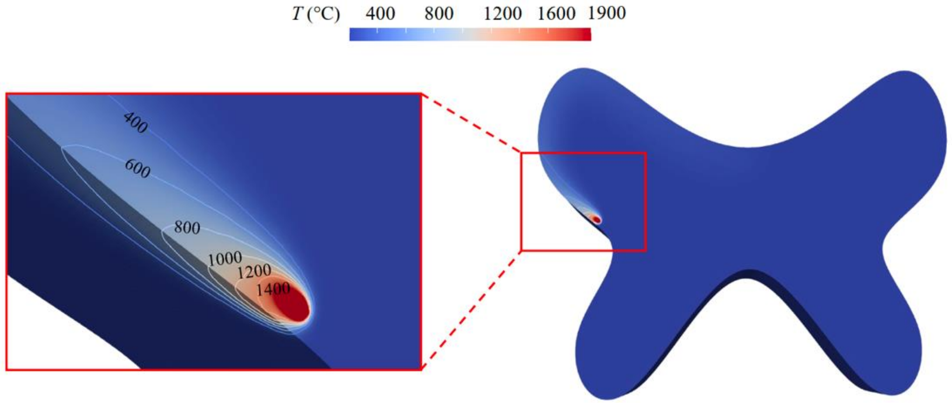
Applications: Academic use cases





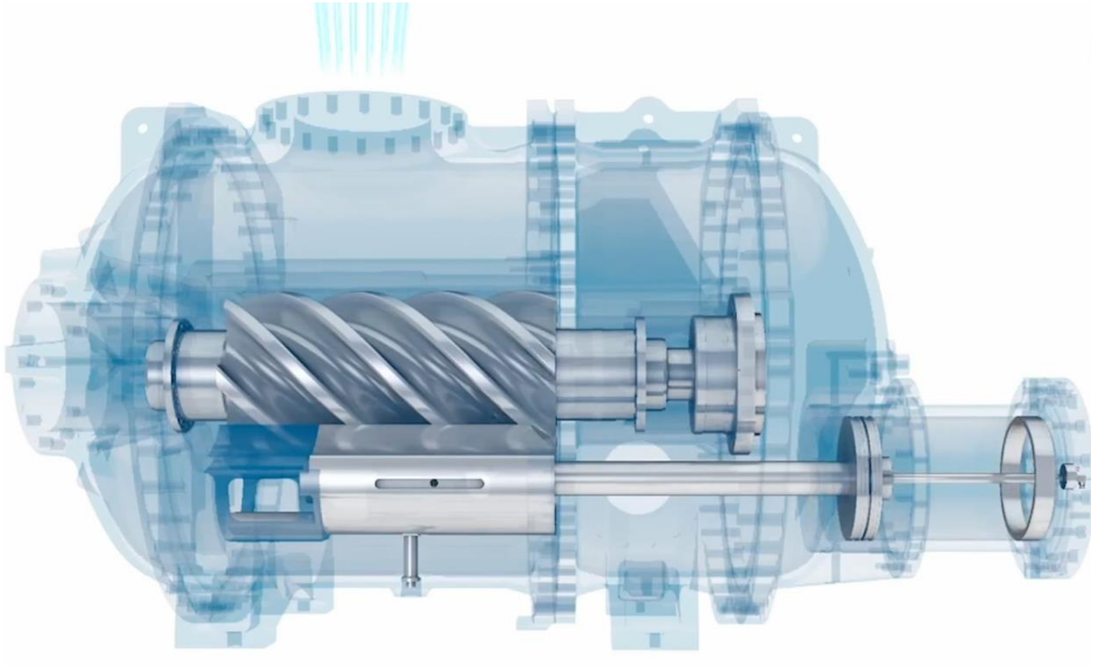






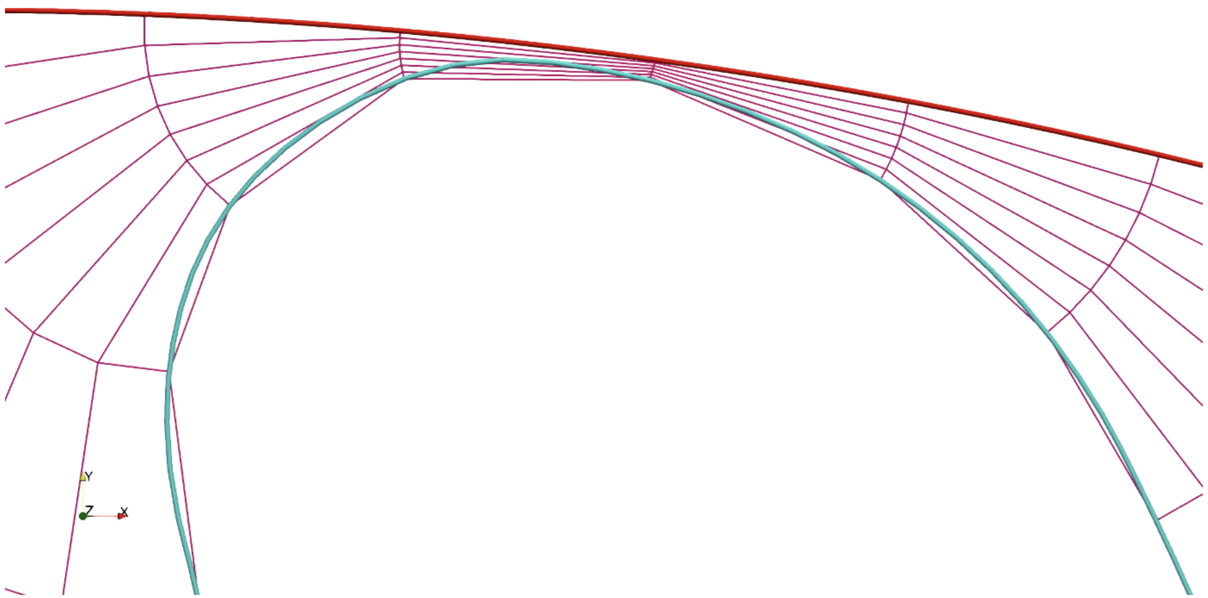
- Curve/surface reparameterization while keeping the geometry;
  - Curve – metal additive manufacturing
  - Surface – VR (Matthias)

Yang, Y., Ji, Y. Möller M, Ayas C. Computational Efficient Process Simulation of Geometrically Complex Parts in Metal Additive Manufacturing, Submitted.



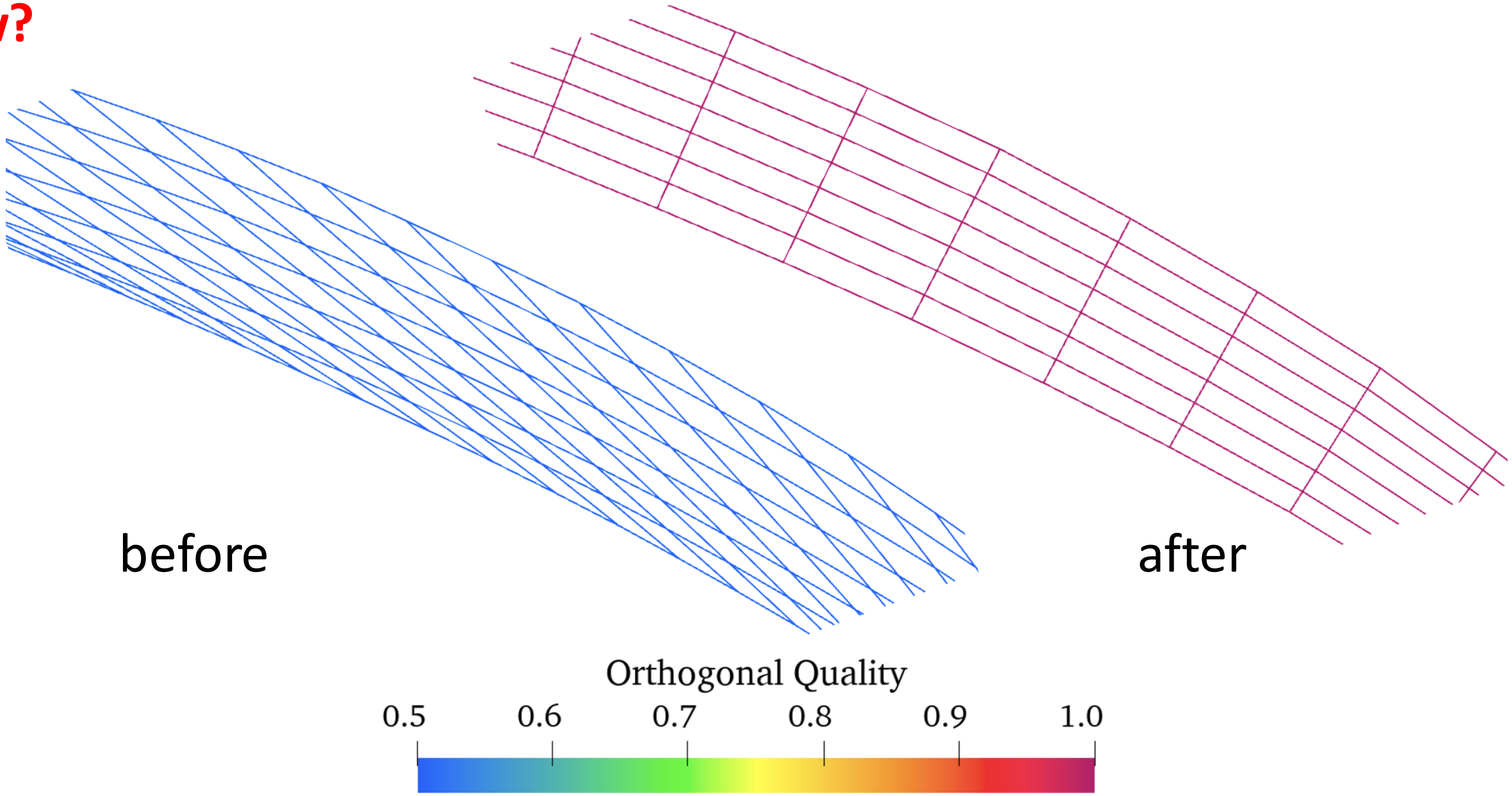
Rotary twin-screw compressor

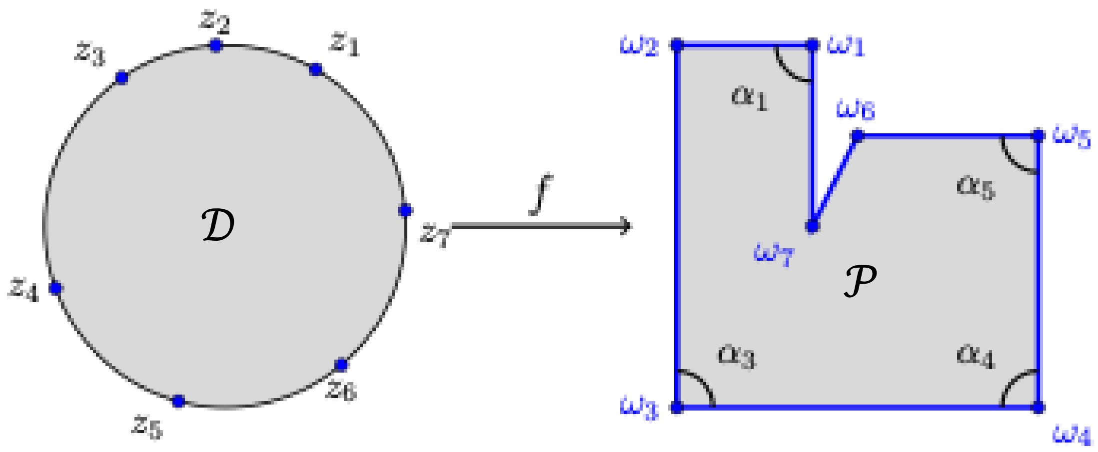
- **Structured mesh generation** is a crucial preprocessing step in the simulation-based analysis of twin-screw machines.
- However, the existing mesh generators typically produce only **linear meshes with straight-sided cells**;
- **Analysis-suitable**, **high-order NURBS parameterizations.**



Source: https://www.gascompressors.co.uk/technologies/oil-floodedscrew-compressor/

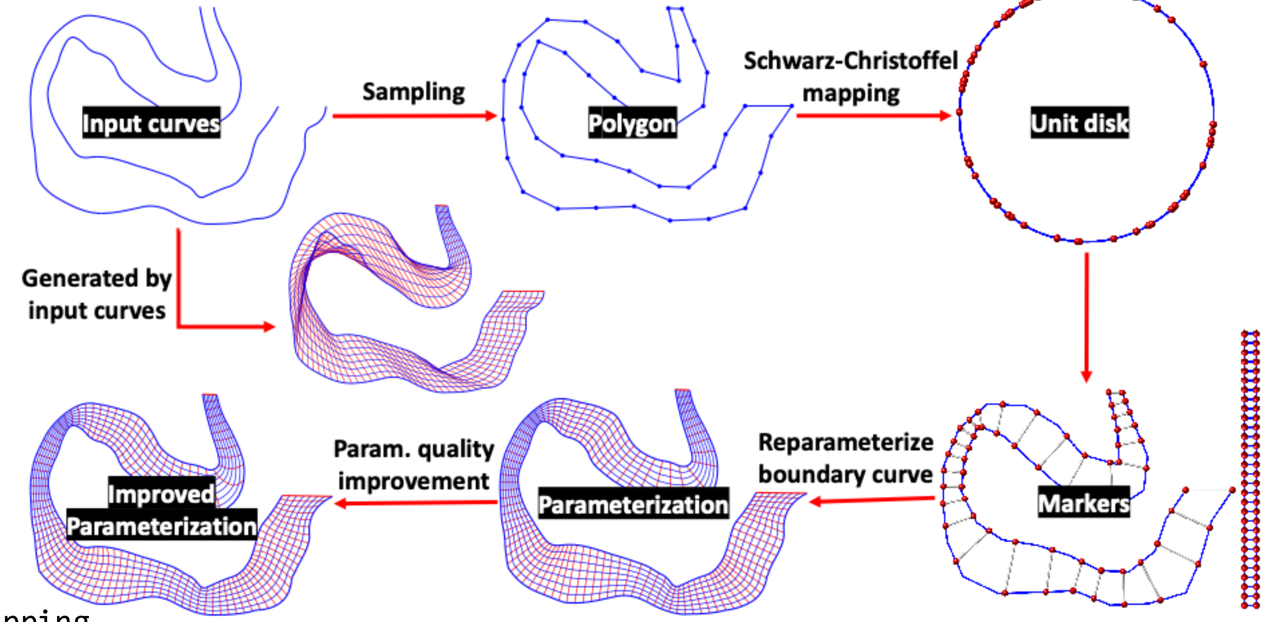
- **Parameter speed** of the boundary curves significantly affects the mesh quality;
- Mesh quality is greatly improved by using the **boundary reparameterization technique.**
- **So, how?**





- **Riemann mapping theorem**: $\exists$ analytic function f **with non-zero derivative** such that $f(\mathcal{D}) = \mathcal{P}$.
- **Schwarz-Christoffel formula**

$$f(z) = f(z_0) + C \int_{z_0}^{z} \prod_{j=1}^{n} \left(1 - \frac{\zeta}{z_j}\right)^{\beta_j} d\zeta$$



- Solving the Schwarz-Christoffel parameter problem for $\{z_j\}$ numerically allows us to compute sets of markers on the two opposite curves that can be used to reparametrize one curve w.r.t. the other.
- Solving the parameter problem is far from easy, **the CRDT algorithm**[1] is adopted and implemented.

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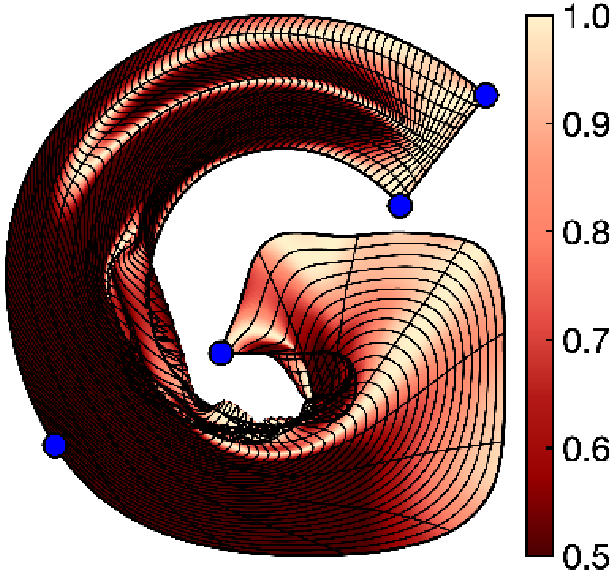
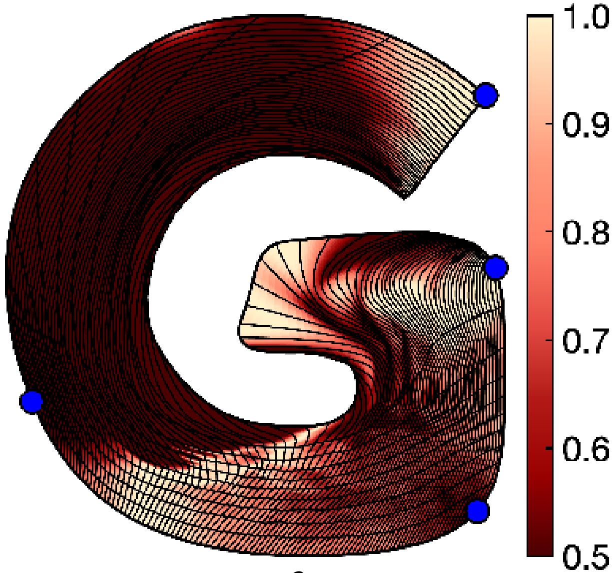
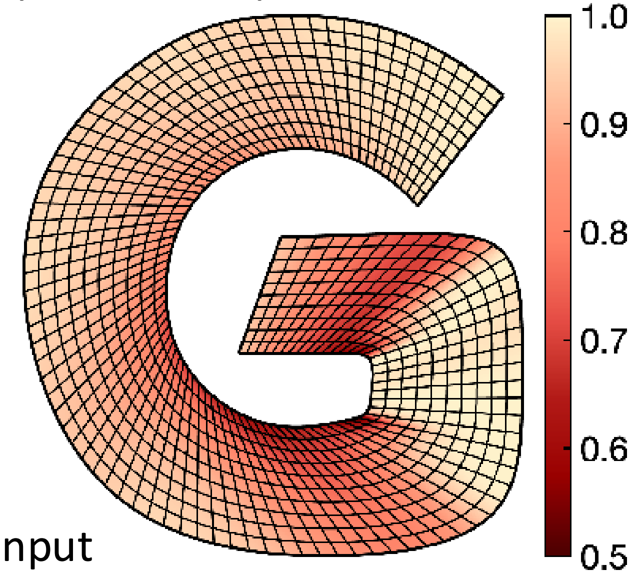
1. Driscoll, T. A., & Vavasis, S. A. (1998). Numerical conformal mapping using cross-ratios and Delaunay triangulation. SIAM Journal on Scientific Computing, 19(6), 1783-1803.



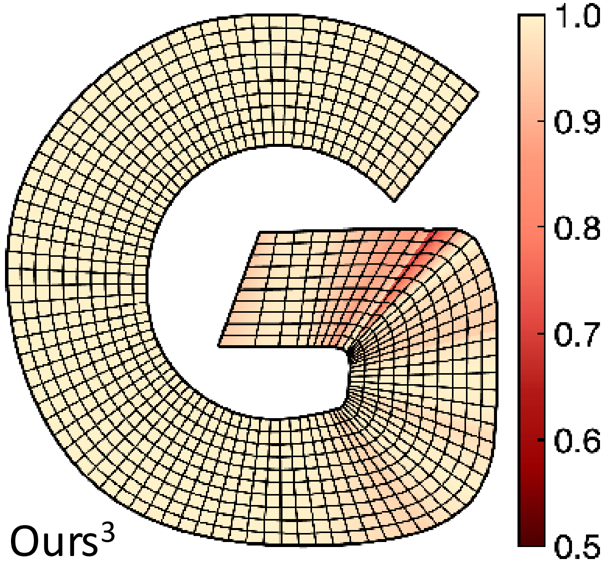
Optimal Transport[1]



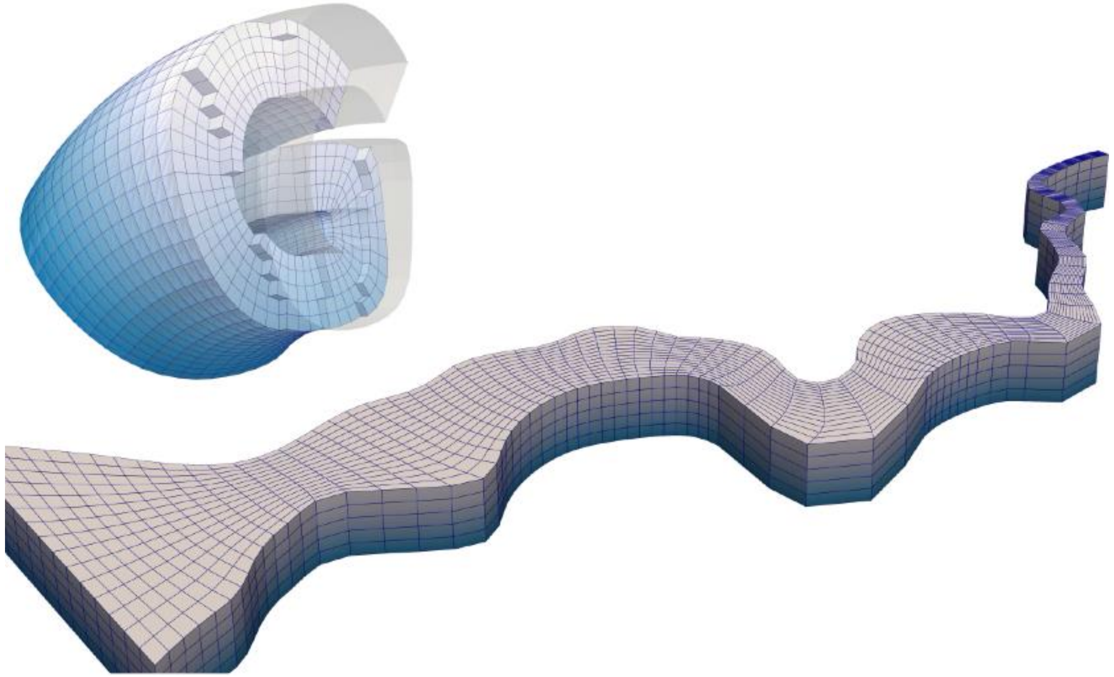
Deep learning[2]



Input

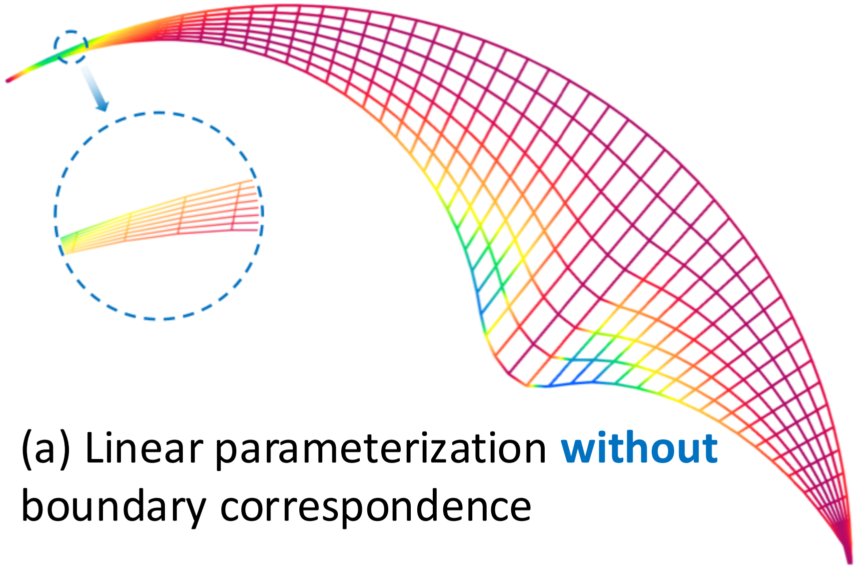


Ours[3]

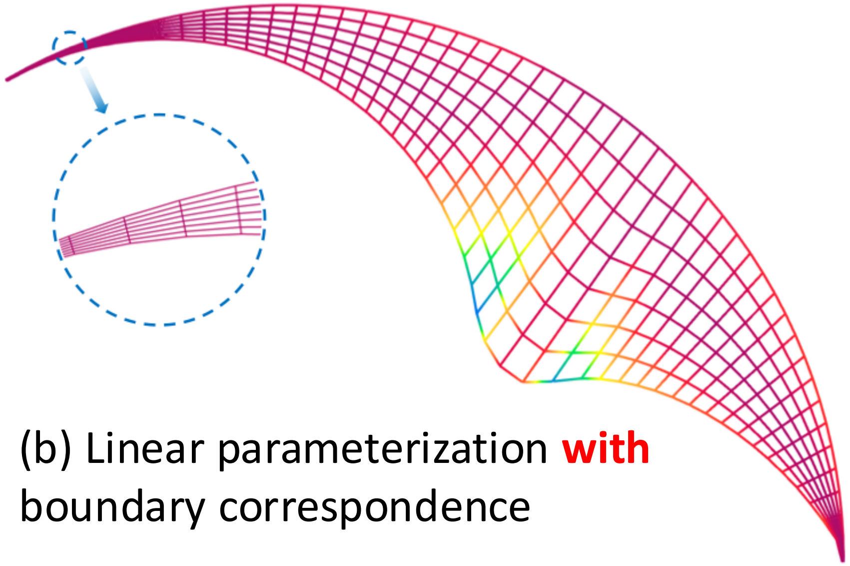


1. Zheng, Y., Pan, M., & Chen, F. (2019). Boundary correspondence of planar domains for isogeometric analysis based on optimal mass transport. Computer-Aided Design, 114, 28-36.
2. Zhan, Z., Zheng, Y., Wang, W., & Chen, F. (2023). Boundary Correspondence for Iso-Geometric Analysis Based on Deep Learning. Communications in Mathematics and Statistics, 11(1), 131-150.
3. Ji, Y., Möoller M., Yu Y., & C. Zhu. (2024). Boundary parameter matching for isogeometric analysis using Schwarz–Christoffel mapping. Engineering with Computers, 1-19.

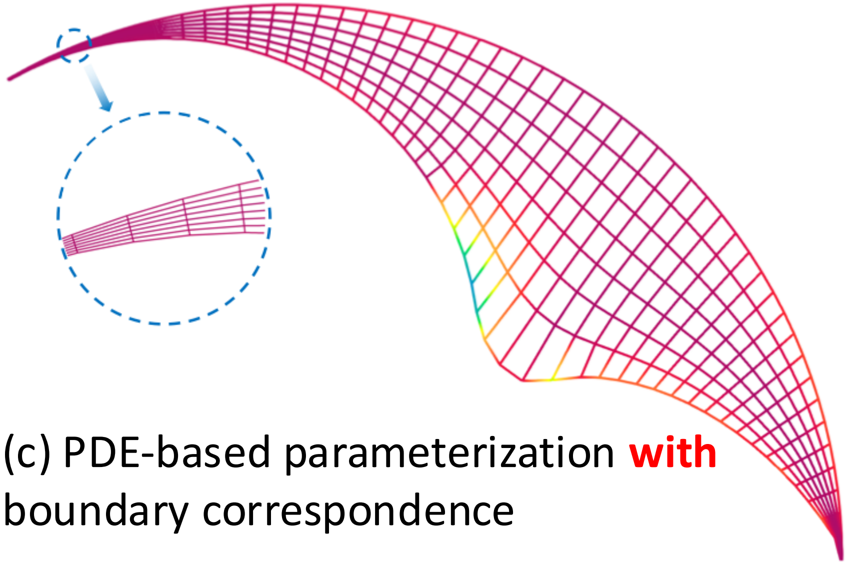
Comparison of different parameterization methods



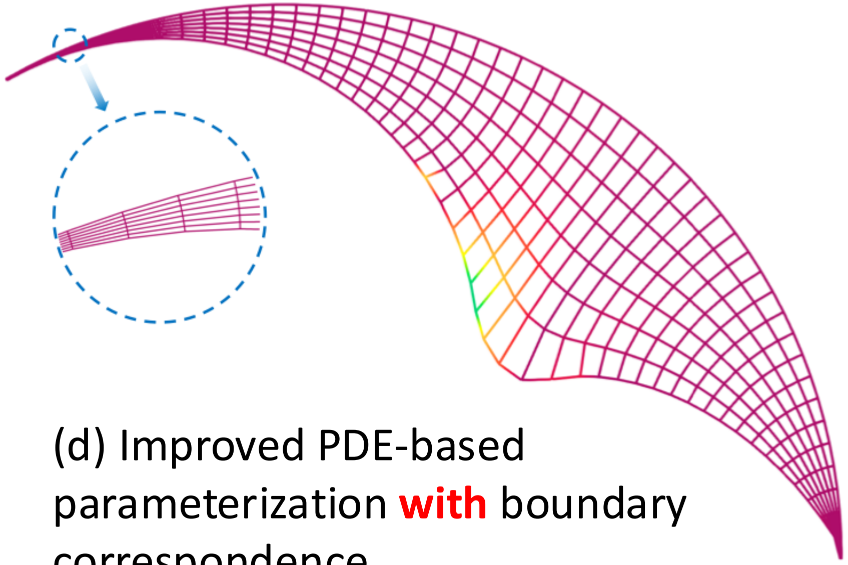
(a) Linear parameterization **without** boundary correspondence



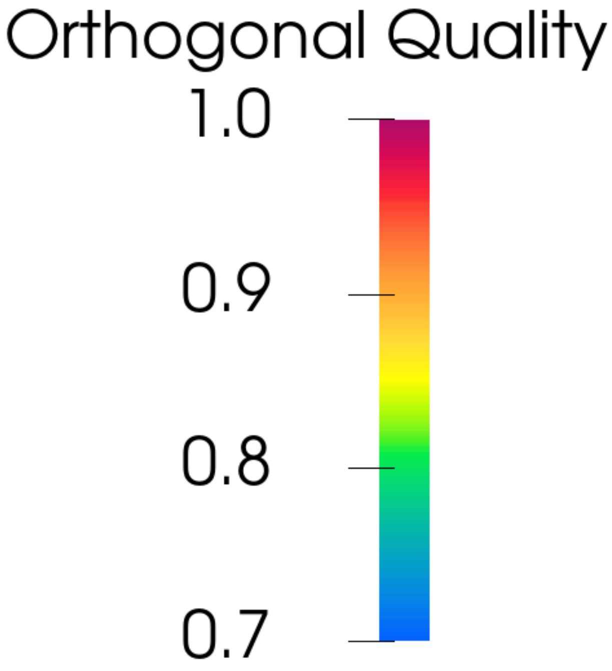
(b) Linear parameterization **with** boundary correspondence



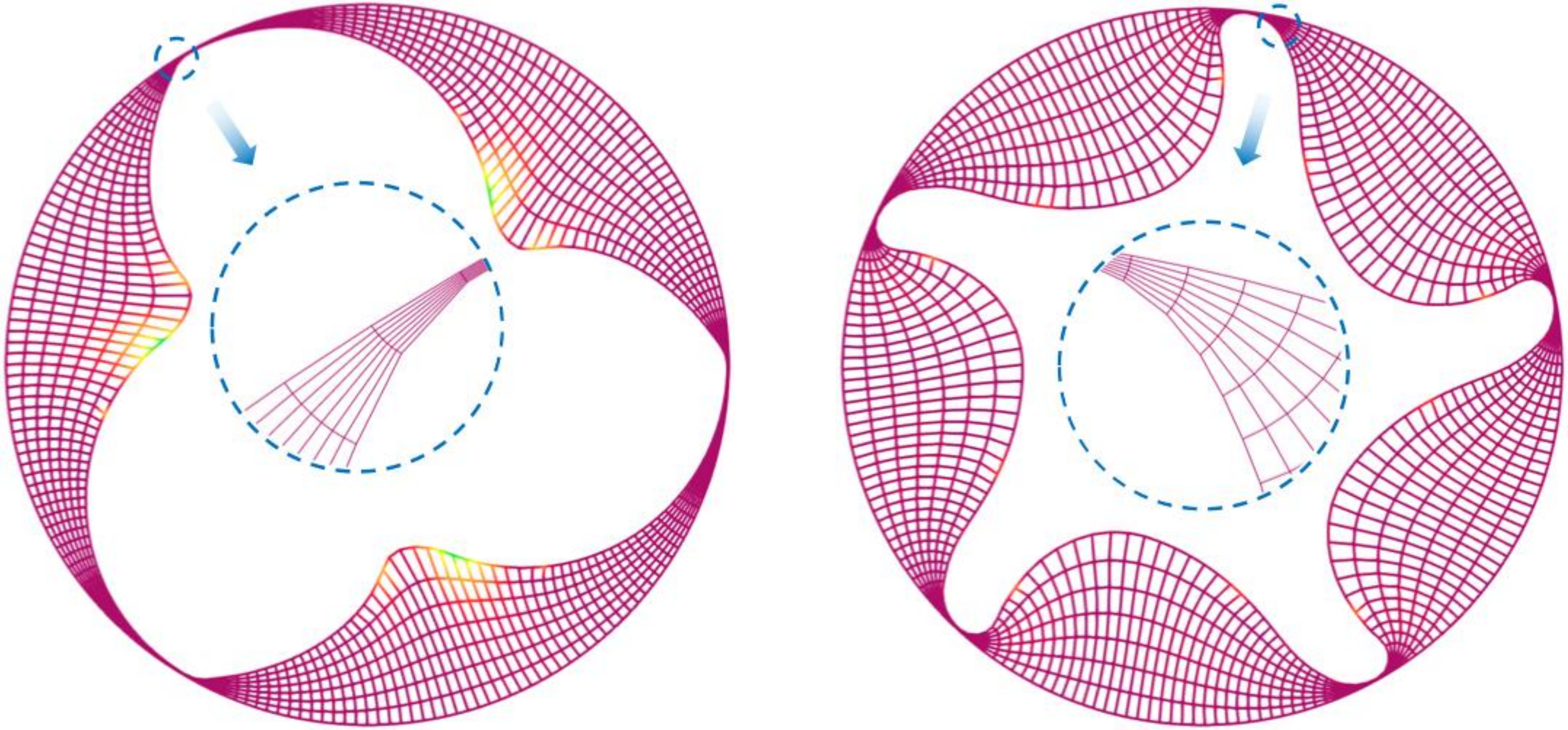
(c) PDE-based parameterization **with** boundary correspondence



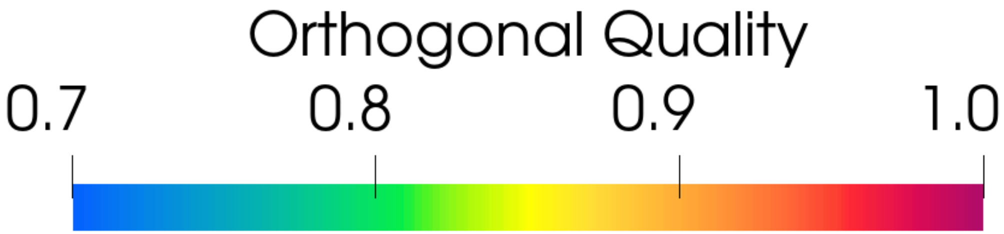
(d) Improved PDE-based parameterization **with** boundary correspondence



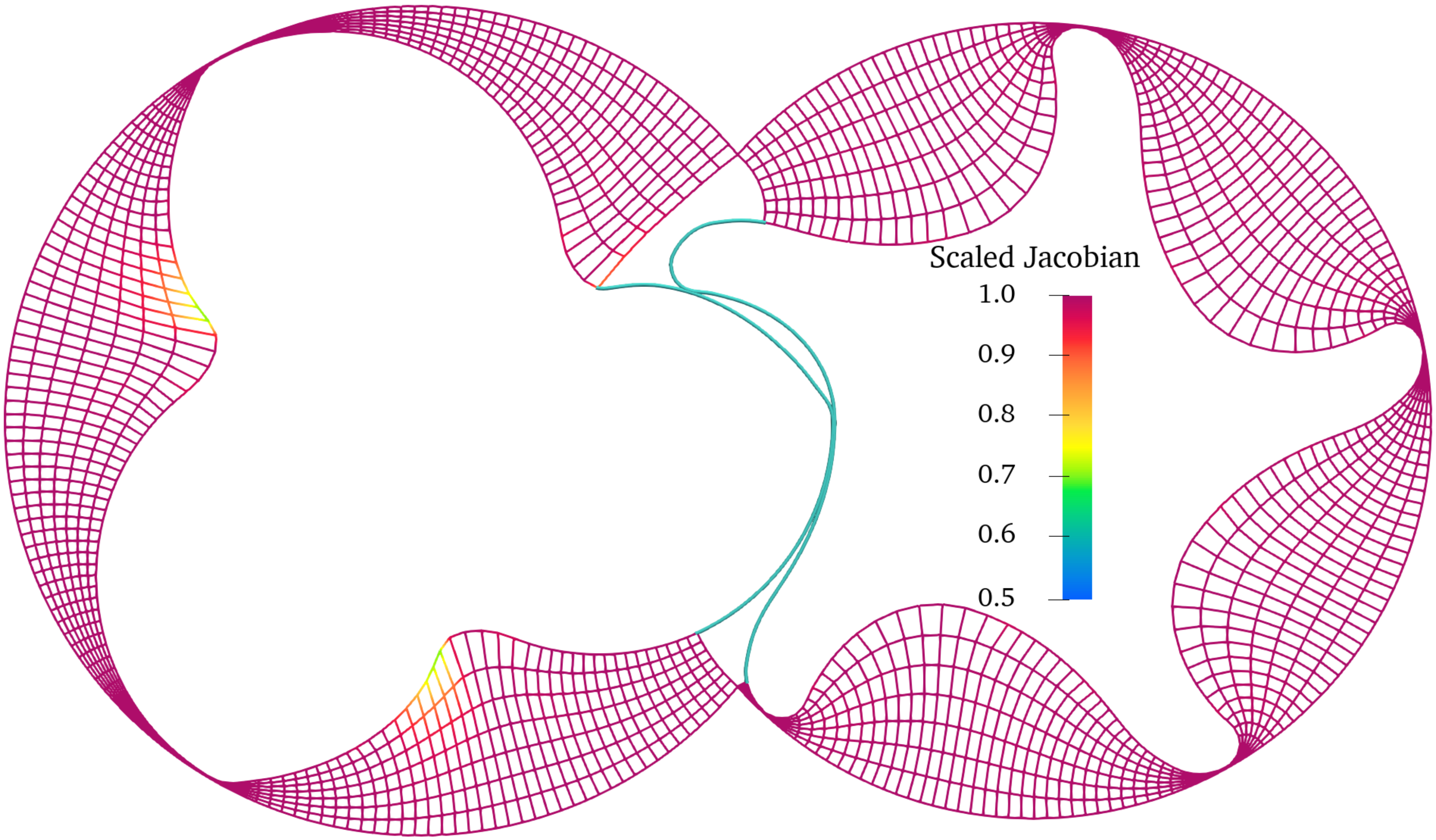
Base parameterizations for main/gate rotors



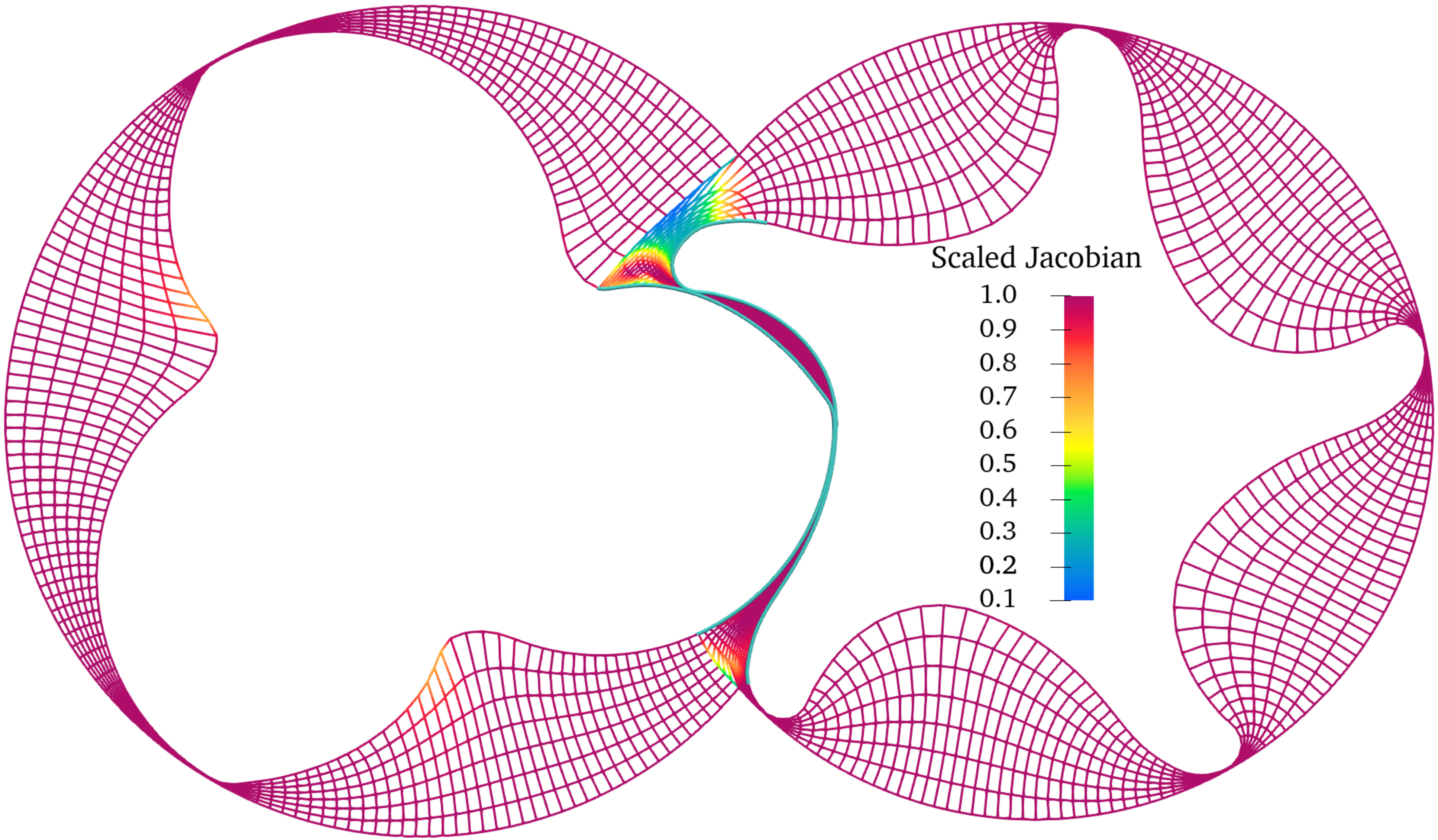
- Obtain the complete base parameterization for each rotor by rotating the one-lobe geometry
  - using inherent symmetry of the geometry



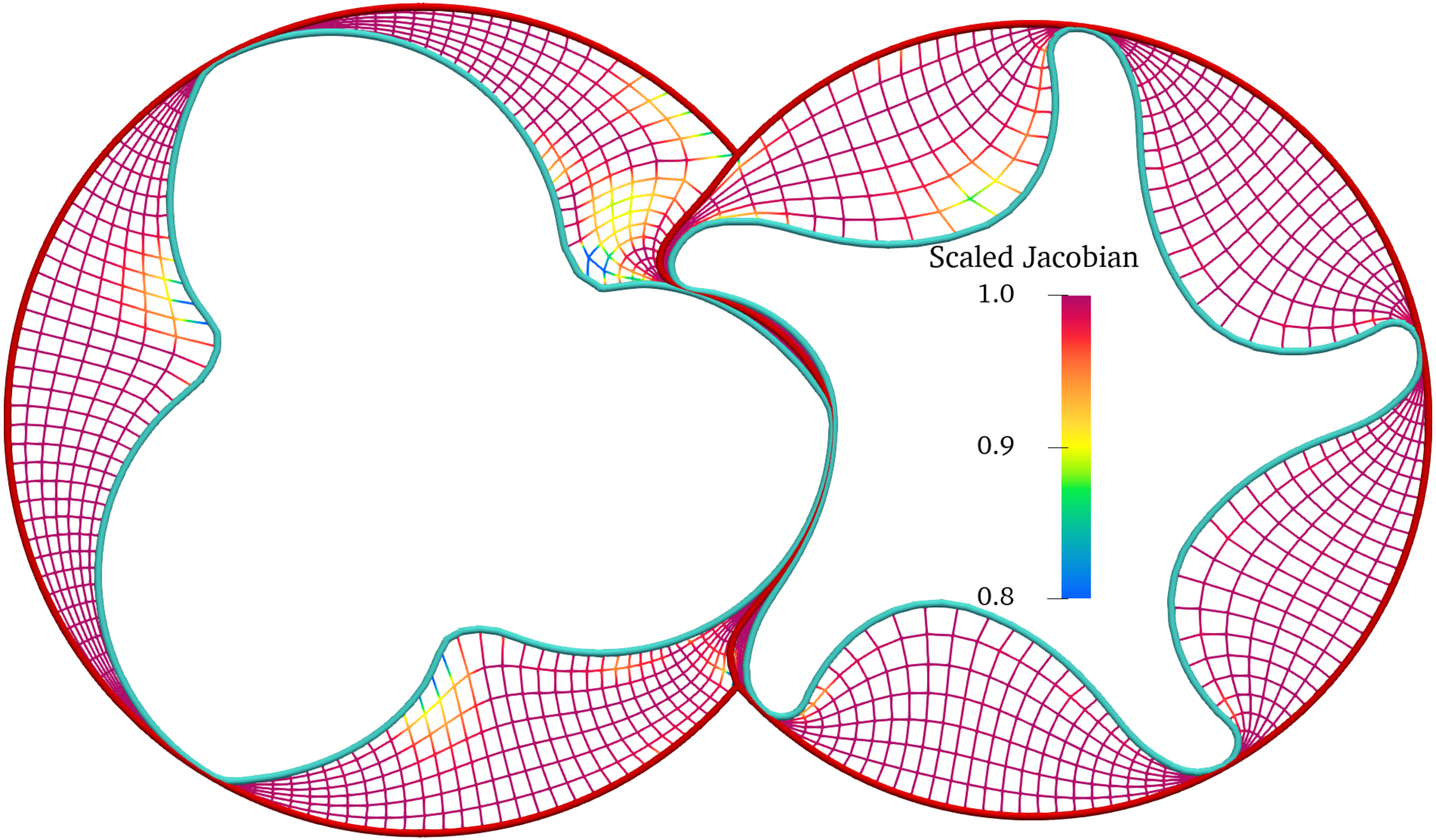
Splitting of the base parameterization at CUSP points



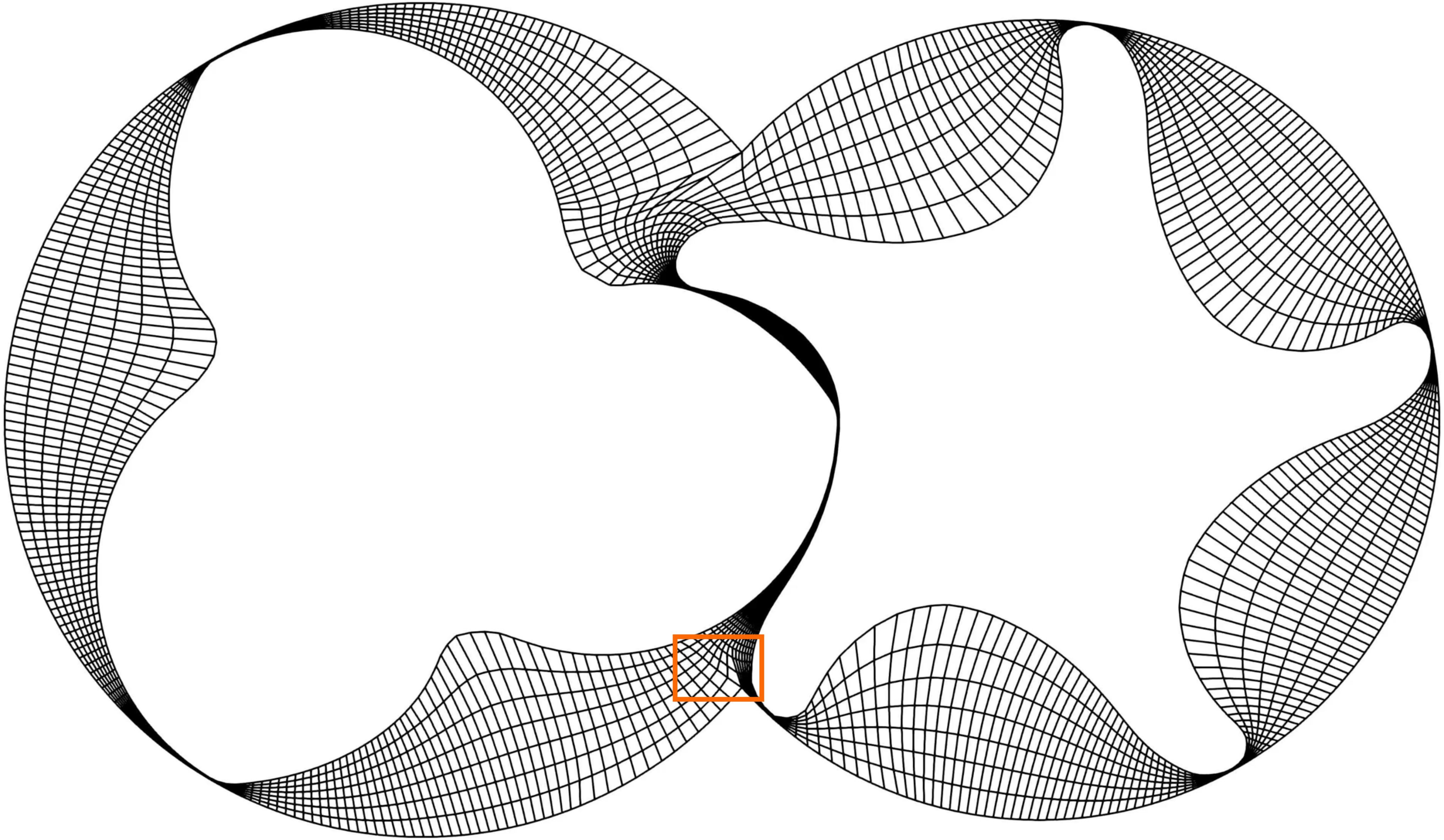
Resulting mesh without smoothness improvement



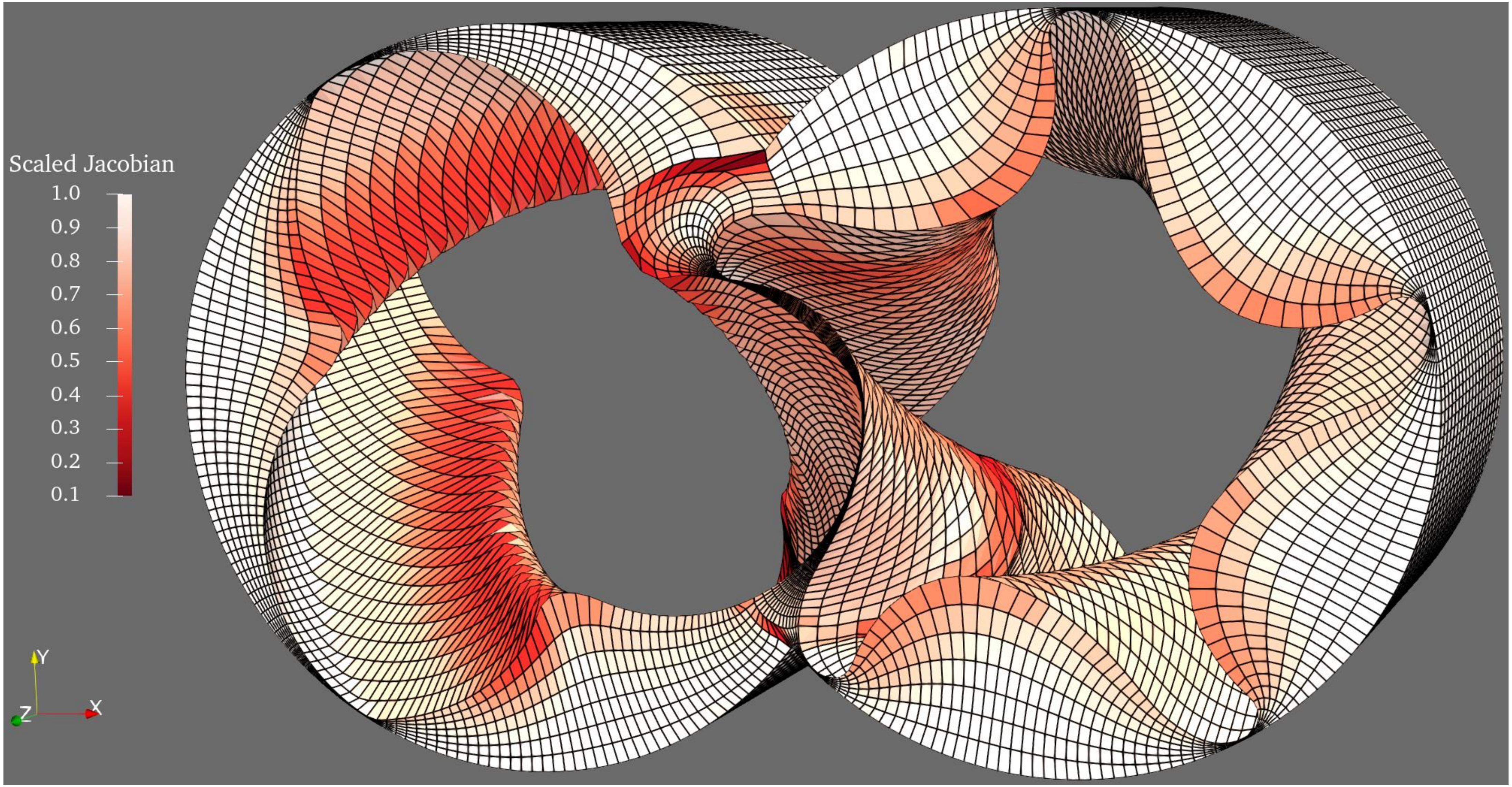
Resulting mesh with smoothness improvement



Resulting mesh with smoothness improvement



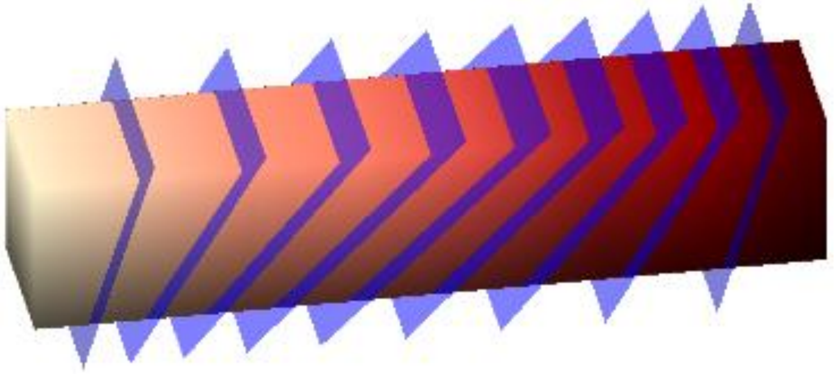
Generated structured hexahedral mesh

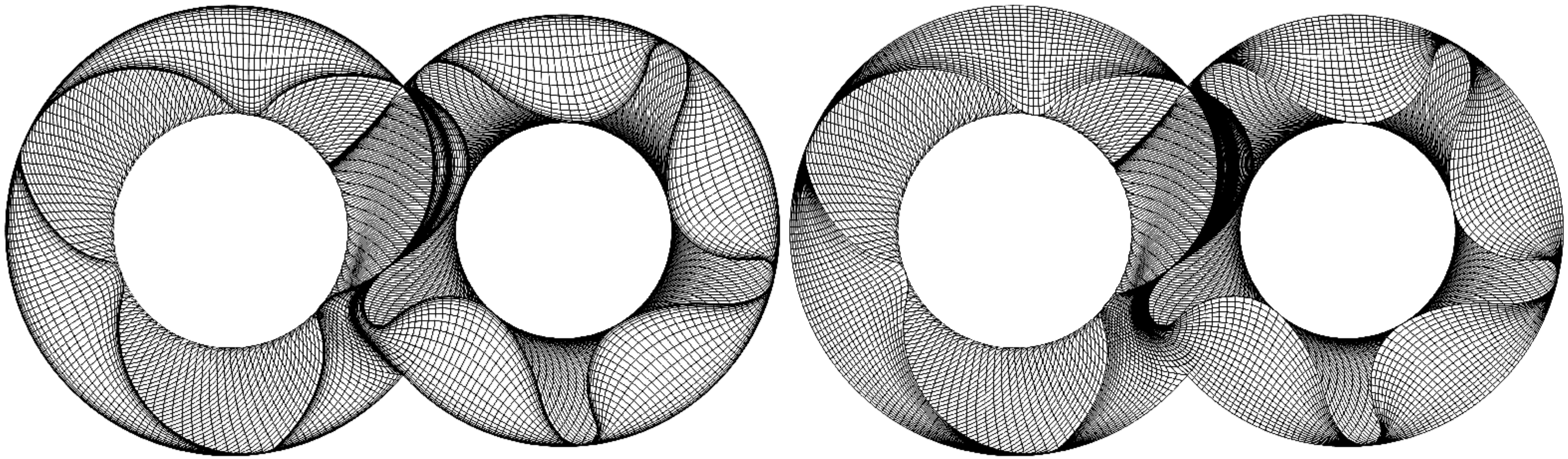


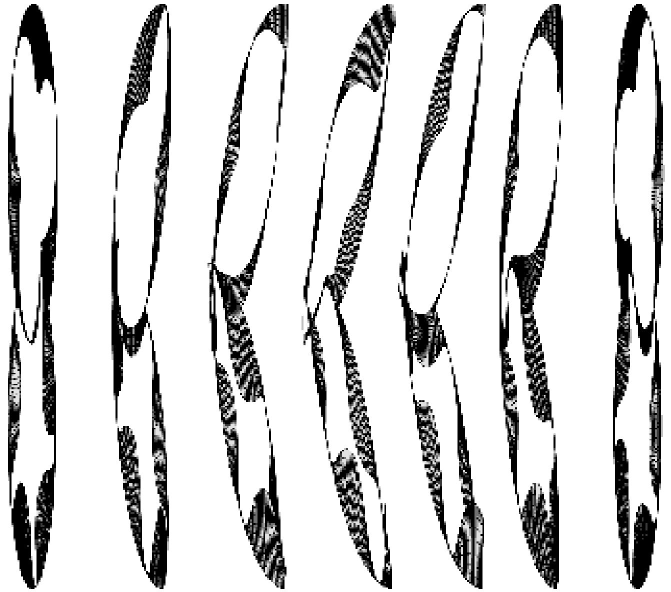
- Boundary layer mesh

$$\begin{cases} \xi = \hat{\xi} \\ \eta = \dfrac{tanh\left(\alpha(2\hat{\eta}-1)\right)}{2\,tanh(\hat{\eta})} + \dfrac{1}{2} \end{cases}$$

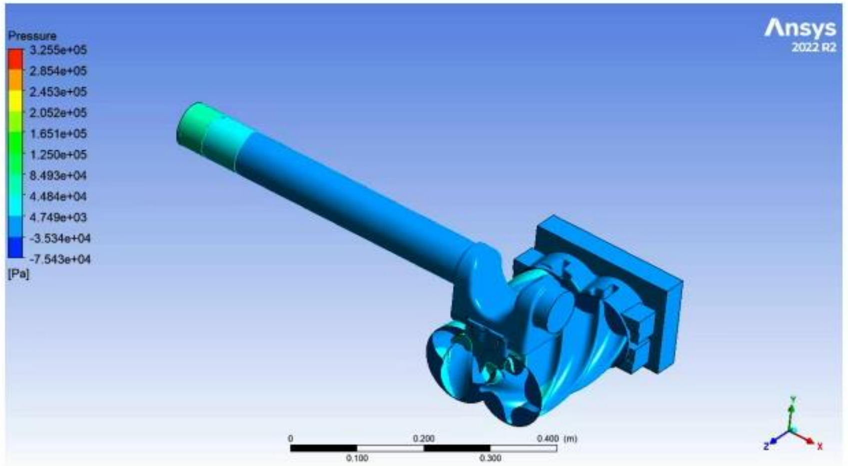
- Flow-aligned discretization

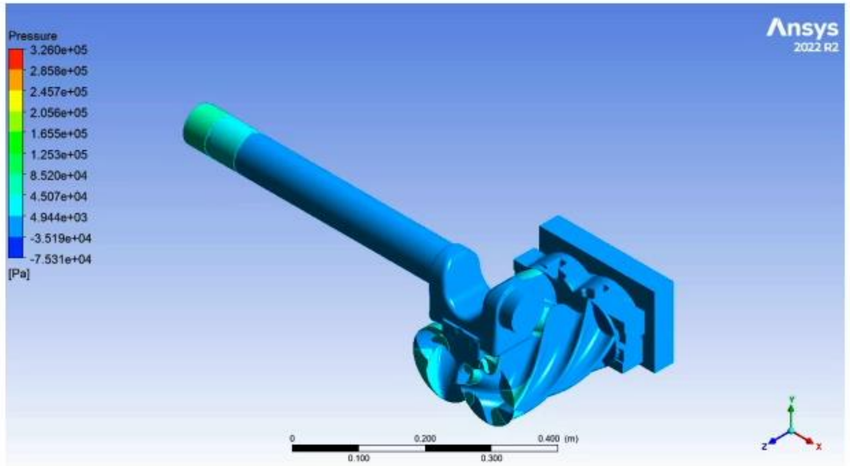


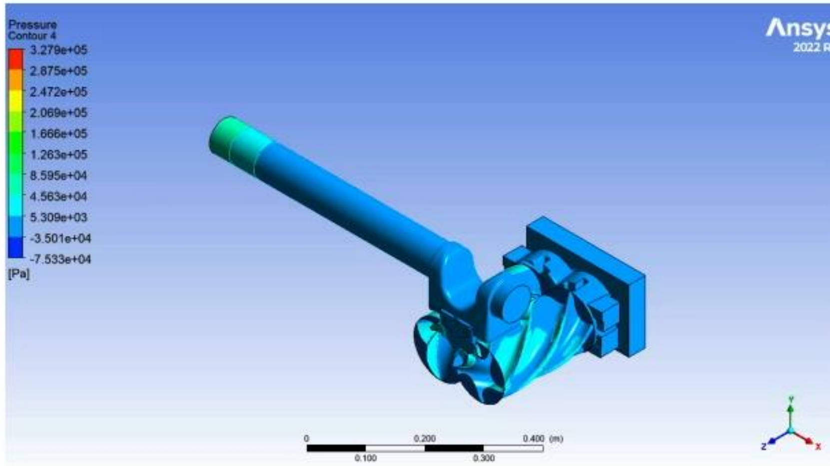




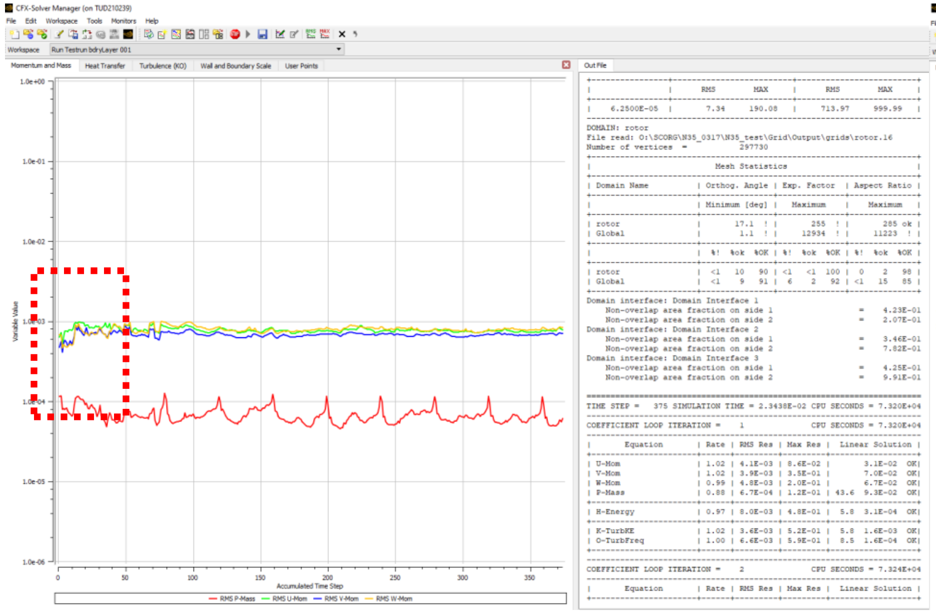
Simulation results using ANSYS CFX



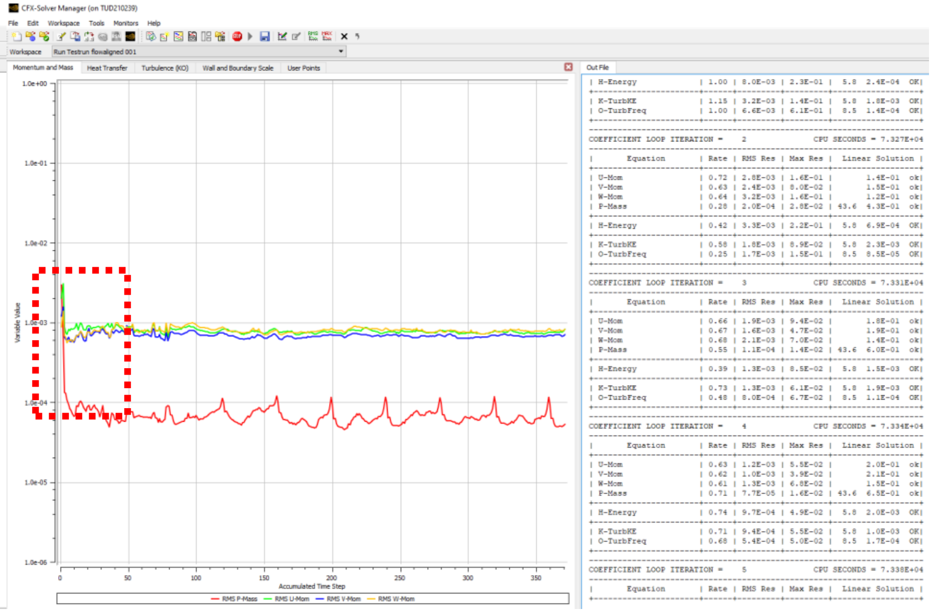




SCORG ™

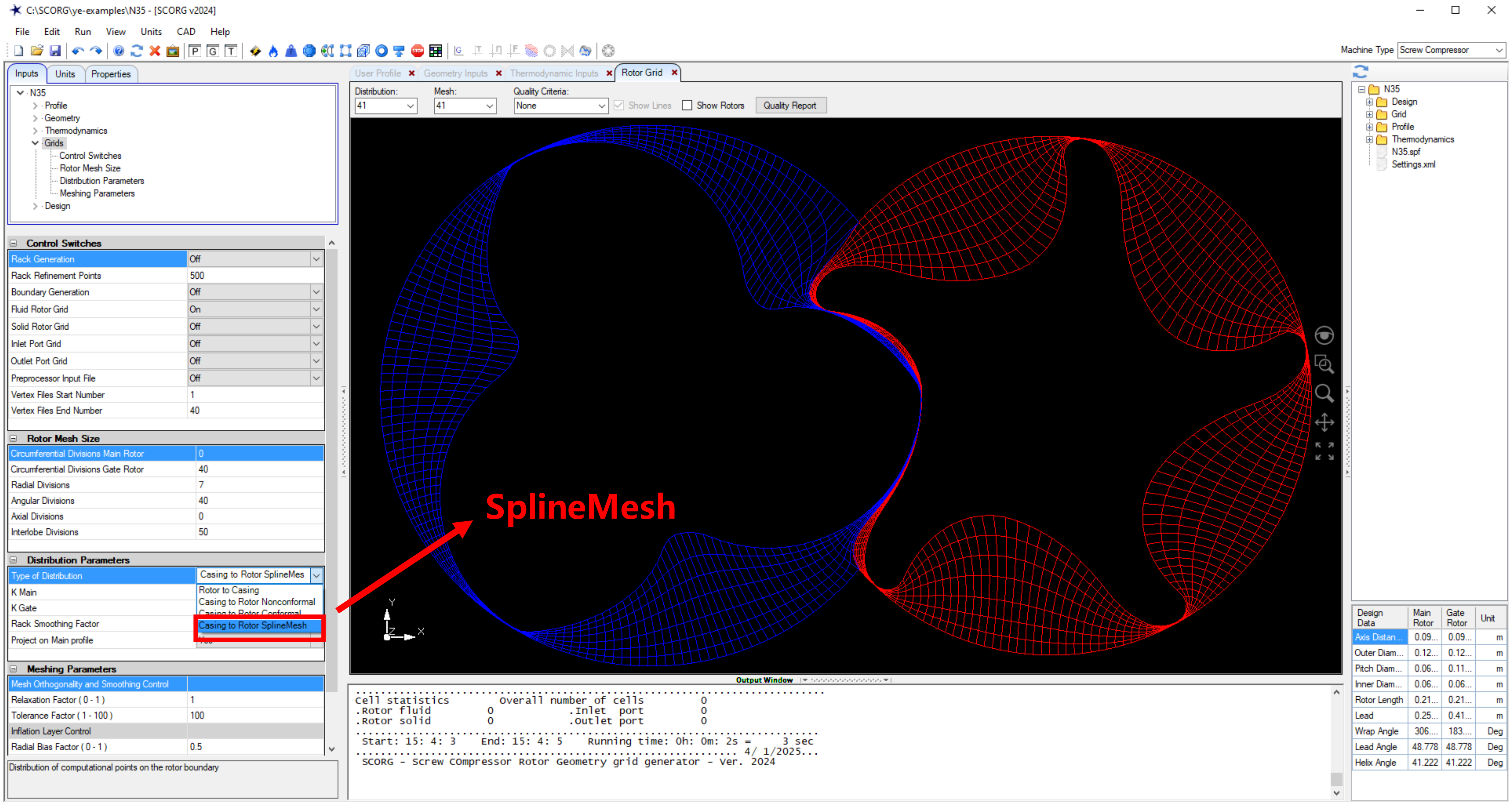


Boundary layer mesh



Flow-aligned mesh

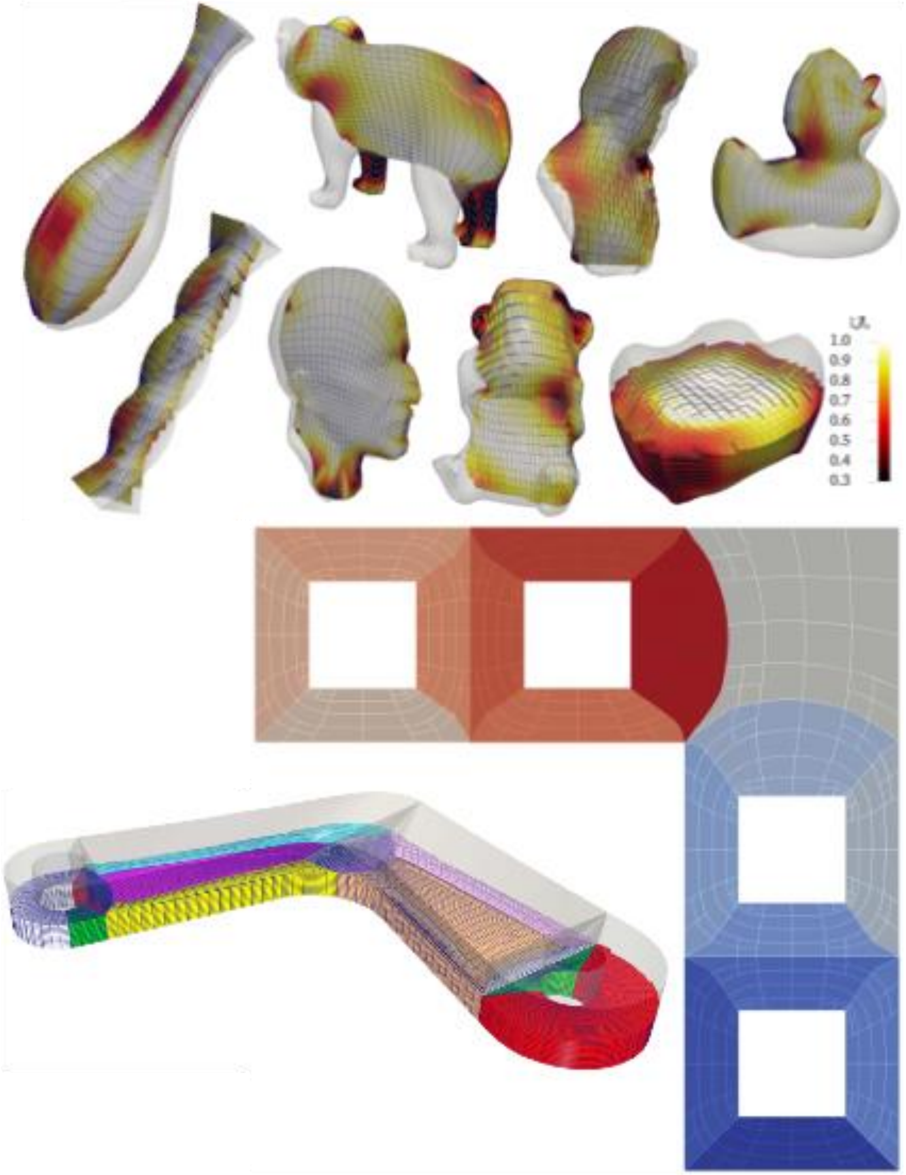
Integration into SCORG™



Outline

- **G+Smo offers** three major classes of **parameterization methods**;
- Demonstrates **improved robustness and efficiency** over existing methods;
- Applicability in both **academic** and **real-world industry scenarios**;

- Future Work:
  - Integrating newly developed and ongoing methods into G+Smo for broader usability;
  - User-friendly **graphical user interface**.



Many thanks for your attention!

Q&A

If you are interested in my research, please feel free to contact me! ;-)

- **Email:** y.ji-1@tudelft.nl
- **GitHub:** jiyess
- **Homepage:** https://jiyess.github.io/

Preconditioned Anderson acceleration solver